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Hybrid coordination of multi-agent networks with hierarchical leaders

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A B S T R A C T

In this paper, the hybrid coordination problem of multi-agent networks with hierarchical leaders is investigated. Firstly, for the case of static major leader with uncoupled follower groups, a necessary and sufficient condition is given for all minor leaders to attain the expected formation and follower groups asymptotically converge to their given desired states. Secondly, for the case of static major leader with coupling follower groups, a sufficient condition on control parameters and the weighted adjacency matrix for the hybrid coordination is provided. Thirdly, for the case of moving major leader with hierarchy delays, a necessary and sufficient condition for the multi-agent network to derive desired performance is also obtained. Finally, some numerical examples are presented to illustrate the theoretical results.

1. Introduction

In the past years, coordination problem of multi-agent systems has attracted many researchers' interest due to its broad applications in many areas such as cooperative control of unmanned aerial vehicles, formation control and swarming behaviors of social living beings. Most of the remarkable and significant results have been focused on first-order or second-order multi-agent systems, see [1–6] for example.

Recently, extensive studies have been conducted for the cooperative control including flocking [7,8], swarming [9], consensus and formation. In [8], a theoretical framework for the design and analysis of distributed flocking algorithms in free-space and presence of multiple obstacles was presented. The aggregation properties of an anisotropic swarm model with an attraction or repulsion function were investigated in [9]. Meanwhile, as an important aspect of coordination control, the leader following consensus has also been studied. In [10], static leader was considered in jointly-connected interaction topologies. Distributed observers were introduced in second-order multi-agent systems to follow a moving leader with unmeasurable velocity in [11]. Much work has been done on the formation stabilization [12–14] and consensus seeking [15–17]. However, the agents of a complex system may present different coordination behaviors, such as some agents acquire formation stabilization and other agents achieve consensus, which can be called hybrid coordination.

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The control of such large scale complex systems poses new challenges that fall beyond the traditional methods. To improve control effectiveness and reliability, multiple leaders are adopted to achieve desired collective behaviors in multi-agent coordination. The multi-leader following problem of second-order multi-agent system with random switching topologies was considered in [18]. In [19], the set tracking of multi-agent systems guided by multiple moving leaders with unmeasurable velocities was investigated. The follower agents aggregated to the convex set spanned by the leaders. Also, target containment control schemes were proposed to make the follower agents stay in the convex set spanned by multiple leaders with random switching topologies [20]. However, in practical situations, animals that travel together in groups may display a variety of fascinating motion patterns. In [21], a hierarchical leadership among flock members of pigeons was investigated and a well-defined hierarchy among flock members was found from data concerning leading roles in pairwise interactions. The authors suggested that hierarchical organization of group flight may be more efficient.

Inspired by [21], the objective of this paper is to study the coordination control for a multi-agent network with hierarchical leaders, which has not been fully investigated up to now. In contrast to the traditional single leader or multiple leaders, hierarchical leaders would be more efficient. In the present work, a major leader, some minor leaders and some followers are assumed in three different levels. Firstly, a necessary and sufficient condition is provided for the hybrid coordination with static major leader and uncoupled follower groups. Secondly, a sufficient condition on control parameters and the weighted adjacency matrix is obtained for the hybrid coordination with static major leader and coupling follower groups. Lastly, for the case of moving major leader with hierarchy delays, a necessary and sufficient condition is presented. Some numerical examples illustrate the proposed results.

The rest of this paper is organized as follows. In Section 2, some preliminaries as well as the problem formulation is presented. In Section 3, the results for the hybrid coordination of hierarchy network with static major leader are proposed. Furthermore, the problem of hybrid coordination with moving major leader and time-varying delay is addressed in Section 4. In Section 5, some numerical examples are given to illustrate the theoretical result. Finally, some concluding remarks are drawn in Section 6.

2. Preliminaries and formulation

In this section, we introduce some preliminary knowledge and our problem formulation for the following discussion.

First, some basic concepts in graph theory are introduced. If each agent is regarded as a node, then a simple graph can be introduced to describe their coupling topology. Let $G = (V, E, A)$ be a weighted digraph of order N, where $V = \{1, 2, \ldots, N\}$ is the set of nodes and $E \subseteq V \times V$ is the set of edges, and the weighted adjacency matrix is denoted by $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ with $a_{ii} = 0$ and $a_{ij} \geqslant 0$ ($a_{ij} > 0$ if there is an edge from node i to node j). The set of all the neighbors of node i is denoted by $N_i = \{j \in V | (i,j) \in E, j \neq i\}$. The graph G is called undirected if $a_{ij} = a_{ji}$, for any $(i,j) \in E$. Its degree matrix $D = diag\{d_1, \ldots, d_N\} \in \mathbb{R}^{N\times N}$ is a diagonal matrix, where $d_i = \sum_{j=1}^{N} a_{ij}$ for $i = 1, \ldots, N$. The matrix $L = D - A$ is called the Laplacian of the weighted graph G, which is symmetric and positive semi-definite for undirected graphs. If the matrix A and B satisfy that $A \geqslant B$, we say that matrix $A - B$ is positive semi-definite.

Here, we study a hierarchical multi-agent network consisting of a major leader (regarded as node 0) and r minor leaders (as r nodes) and N followers (as N nodes). At first, the interconnection relationships among r minor leaders is described as a graph G_l. The Laplacian matrix of graph G_l is denoted as L_l. The N followers is divided into r subgroups, where the subgroup $G_m,\ m = 1, \ldots, r$, consisting of N_m nodes with $N_1 + \cdots + N_m + \cdots + N_r = N$. Without loss of generality, we assume that each subgroup G_m is matched to a minor leader m. Taking account of the major leader and minor leader, we concern a graph $\bar{G}_l$ consisting of graph G_l, vertex 0 and directed edges from some vertices to vertex 0, a graph $\bar{G}_m$ consisting of graph G_m, vertex m and directed edges from some vertices to vertex m. For $\bar{G}_l$, if there is a path in $\bar{G}_l$ from every vertex m in G_l to vertex 0, we say that vertex 0 is globally reachable in $\bar{G}_l$. Similarly, if there is a path in $\bar{G}_m$ from every vertex i in G_m to vertex m, we say that vertex m is globally reachable in $\bar{G}_m$.

The major leader's control signal is not influenced by the other agents and the dynamics of the major leader can be represented by

$$\dot{x}_0(t) = v_0, \tag{1}$$

where $x_0(t) \in \mathbb{R}$ and $v_0 \in \mathbb{R}$ indicates the state and the desired constant velocity of the major leader, respectively. For simplicity, we take the dimension of state vector as one in this paper. Obviously, the corresponding results can be extended to arbitrary dimensions with the Kronecker product properties.

The minor leaders take command and control from major leader or the other minor leaders, a continuous-time model of the r minor leaders is expressed as follows

$$\dot{x}_m(t) = u_m(t), \quad m = 1, \ldots, r, \tag{2}$$

where r represents the number of the minor leaders, $x_m(t) \in \mathbb{R}$ and $u_m(t) \in \mathbb{R}$ denotes the state and control input of the minor leader m, respectively.

Our goal is to let all the minor leaders follow the static major leader in form of certain formation, i.e., $x_m(t) \to x_0 - \bar{x}_m$ as $t \to \infty$, where $\bar{x}_m$ is the desired deviation of the mth minor leader with the major leader.