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Application of game theory on parameter optimization of the novel two-stage Nash nonlinear grey Bernoulli model

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ABSTRACT

In grey prediction modeling, there are three parameters in nonlinear grey Bernoulli model (NGBM), including the power n, the coefficient p and the length of raw data used to construct grey forecasting model. Nash NGBM only optimizes n and p by the iterated elimination of weakly dominated strategies of game theory. To optimize above three parameters, this study develops a two-stage game for NNGBM (abbreviated as two-stage NNGBM). In the first stage, find the Nash equilibrium for each possible game. In the second stage, use Minimax principle to find the optimal left topological sequence which has the best forecasting performance. Then, obtain Nash equilibrium which consists of these three parameters. This study also proves that the traditional GM(1,1), optimal p GM(1,1) and NGBM(1,1) are the special cases of the proposed model. In order to show the feasibility of this research, the proposed method is applied to the forecasting of Taiwan's GDP. The results show that five elements in raw data sequence are optimal topological length for constructing NNGBM in the case of Taiwan's GDP forecasting.

1. Introduction

Nowadays, more than 300 different prediction techniques are widely applied in various academic fields. All techniques could be classified into qualitative and quantitative methods. For instance, qualitative forecasting methods include the Delphi method and the expert system so on. The quantitative forecasting methods include linear regression analysis, neural networks, time series analysis and grey forecasting [1]. Every prediction technique has its unique strength and weakness, so the researchers have to realize the characteristic of each forecasting method in order to maximize the strength of each method.

Deng [2] proposed grey theory more than 30 years ago. The feature of grey forecasting is small quantity of data needed to make satisfactory in dealing with incomplete and uncertain information. Grey theory reduces the randomness of raw data to a monotonic increasing sequence by accumulated generating operation (AGO). The accumulated data fits the solution of first order ordinary differential equation because both have monotonic increasing tendency. Thus, inversed accumulated generating operation (IAGO) on the discrete form of solution can generate a prediction value. Grey forecasting has been successfully applied in social and nature science [3–11].

Traditional grey forecasting uses basic mathematics and its calculation process is easily understood. Also, the achieved prediction accuracy is satisfactory. To enhance the prediction precise of grey forecasting, researchers endeavor to develop

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the hybrid grey model to further increase the prediction precision, including Grey-Fuzzy model [8], Grey–Taguchi model [12], Modified Grey GM (0, 1|sin) model of oscillation sequence [13], Xu and Liu [14] adopted matrix based on cross-sectional data, and the mean sequence of row vector GM(1,1) model and so on. Those works substantially improve forecast ability of grey theory and advance grey theory to more powerful tool in forecasting science.

The intrinsic of GM(1,1) is linear model. The solution is the exponential function and the rate of change is moderate. In order to increase the fitting ability of raw data, Chen [15,16] developed nonlinear models based on Bernoulli differential equation which is called NGBM. Hereafter, some researches explored how to obtain the optimum parameters of NGBM. Scrutinizing the NGBM, there are three underdetermined parameters which are power n, background value p and the length of raw data sequence. Chang [3] proposes jump-p GM(1,1), Wang et al. [9] proposed a combination method of optimization of the background value and optimization of the initial item and Zhou et al. [17] used particle swarm optimization to obtain the optimum parameters of Nash NGBM. Chen et al.[18] obtained the optimum parameters of Nash NGBM. Through optimization of parameters of grey forecasting model, the forecasting precision is greatly increased.

In fact, there are three parameters in NGBM(1,1), including the coefficient p, the power n and the length of time sequence for constructing grey forecasting model. All above articles do not consider the three parameters simultaneously. For this reason, this study develops a two-stage game to make three parameters endogenous.

Through literature reviewing and surveying, grey theory and grey hybrid model are well developed and well applied in various areas. Up to now, there is few discussion of game theory combined with grey theory and this is reason why this research is arouse. The motivation of conducting this research arose. Before the game theory begins, this study converts NGBM(1,1) into a game form that includes players, strategy and payoff function in a normal-form game. In the first stage, find Nash equilibrium for each player in all possible game. In the second stage, players choose an optimal game which has minimum cost function by Minmax principle.

In short, the power n and the coefficient p are endogenous in the first stage. The length of time sequence for constructing grey model is endogenous in the second stage. After the game end, Nash equilibrium consists of above three parameters. The traditional GM(1,1), optimal p GM(1,1) and NGBM(1,1) are special cases of the novel proposed model. This study names the novel model two-Stage Nash NGBM(1,1). Finally, the proposed model is applied to the forecasting of Taiwan's GDP for years 2012 and 2013. The results may serve valuable information to decision makers in the development of their future economic policies.

This paper is organized as follows. Section 2 introduces NNGBM(1,1) and develops a two-stage game for grey forecasting model. Section 3 compares the forecasting performances among grey models. A case study is proposed in Section 4 to examine the novel model. Conclusion is present in Section 5.

2. Methodology

The procedures of deriving a two-stage game for Nash NGBM(1,1) are described below:

2.1. Nash nonlinear grey Bernoulli model, NNGBM

Step 1: Assume the original sequence is

$$X^{(0)}(1,m) = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(m)\}, \quad m \geq 4 \tag{1}$$

Step 2: The partial sequences which are called the left topological sequence are defined as:

$$\begin{aligned}
&X^{(0)}_{1.m} = \{x(1), x(2), \ldots, x(i), \ldots, x(m-1), x(m)\} \\
&X^{(0)}_{2.m} = \{x(2), x(3), \ldots, x(i), \ldots, x(m-1), x(m)\} \\
&\vdots \\
&X^{(0)}_{i.m} = \{x(i), x(i+1), \ldots, x(m-1), x(m)\} \\
&\vdots \\
&X^{(0)}_{m-3.m} = \{x(m-3), x(m-2), x(m-1), x(m)\} \\
&, 1 \leqslant i \leqslant m-3, m \geqslant 4
\end{aligned} \tag{2}$$

where raw matrix $X^{(0)}_{i.m}$ represents the non-negative original time series data.

Step 3: Using one time accumulated generation operation (1-AGO) to construct $X^{(1)}_{i.m}$, namely

$$\begin{aligned}
X^{(1)}_{i.m} &= \{x^{(1)}(i), x^{(1)}(i+1), \ldots, \ldots, x^{(1)}(m-1), x^{(1)}(m)\}, \\
&= \left\{\sum_{i}^{i+0} x^{(0)}(t), \sum_{i}^{i+1} x^{(0)}(t), \ldots, \sum_{i}^{m-1} x^{(0)}(t), \sum_{i}^{m} x^{(0)}(t)\right\}, \quad \text{where, } 1 \leqslant i \leq m-3, m \geqslant 4
\end{aligned} \tag{3}$$