Commun Nonlinear Sci Numer Simulat 27 (2015) 228–236

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Network coherence in the web graphs

Qingyan Ding, Weigang Sun *, Fangyue Chen

Department of Mathematics, School of Science, Hangzhou Dianzi University, Hangzhou 310018, China

ARTICLE INFO

Article history:
Received 9 August 2014
Received in revised form 28 February 2015
Accepted 24 March 2015
Available online 28 March 2015

Keywords:
Network coherence
Laplacian spectrum
Web graphs

ABSTRACT

Network coherence is used to characterize the consensus dynamics with additive stochastic disturbances and can be described by Laplacian spectrum. In this paper, we mainly obtain the scalings of network coherence in the web graphs with a special feature that its fractal dimension is infinite. We then investigate the relationship between the scalings and fractal dimension. Based on the structures of web graphs, we obtain the relationships for Laplacian matrix and Laplacian eigenvalues between web graphs and their corresponding equilateral polygons. We also obtain analytical expressions for the sum of the reciprocals and square reciprocals of all nonzero Laplacian eigenvalues. Finally we calculate first and second order coherence and see that the scalings of network coherence with network size N are N and N^3, which shows that the scalings are not related to the fractal dimension of web graphs. In addition, the scalings of network coherence in web graphs are larger than those performed on some fractal networks.

1. Introduction

Complex networks have been proved as an important tool for revealing the structures and functions of natural and man-made systems with special focus on some attributes, such as small average path length, high clustering coefficient, and power-law degree distribution [1–3]. Among of diverse network models, deterministic networks [4–6] have attracted increasing attention because they have an advantage with precise formulations on the above-mentioned attributes. In general, deterministic networks are defined against random networks where new nodes are connected to existing nodes with a certain probability. Fractal networks [7–9] constructed by fractal structures belong to a family of deterministic networks, the main issues that require consideration are network modeling [10], random walks [11,12], and calculations of Laplacian spectrum [13].

Recently coherence in deterministic networks has been an emerging topic with aim on the interplay between network coherence and network topological structures [14–16]. Network coherence characterized by Laplacian spectrum of a network is used to measure the consensus dynamics with additive stochastic disturbances. For the consensus issue in the multi-agent systems, a key task is to design appropriate distributed consensus algorithms. There are many broad applications, such as satellite formation flying, cooperative unmanned vehicles, and sensor networks [17–19]. For the coherence of a network, the research goal is to study the effect of topological measurements on the coherence. Young et al. derived analytical expressions for first-order coherence in rings, path graphs and star graphs [14]. Patterson and Bamieh studied first and second order coherence in fractal networks and found that the scalings of network coherence are related to fractal dimension

* Corresponding author. Tel.: +86 571 86878594.
E-mail address: wgsun@hdu.edu.cn (W. Sun).

http://dx.doi.org/10.1016/j.cnsns.2015.03.011
1007-5704/

[15,16]. In our recent published paper [20], it has been shown that the scalings of network coherence in a family of treelike networks do not depend on their fractal dimension, which shows that it is not a unique factor on the coherence behavior.

In the above-mentioned work [15,16], the fractal dimensions of studied fractal networks are finite. A natural question arises, what are the scalings of coherence in a network with infinite fractal dimension. Therefore, we investigate the network coherence in the web graphs with infinite fractal dimension, which are defined as joining the pendant points of a helm to form a cycle and then adding a single pendant edge to each vertex of this outer cycle [21]. Here we propose a method to obtain exact expressions of first and second order coherence and obtain their scalings with network size N are N and N^3. Obviously the scalings are not relevant to the fractal dimension. We also find that the scalings of web graphs are larger than those of other studied networks. Finally we investigate the effect of network parameter on the coherence behavior.

The rest of this paper is organized as follows. Section 2 gives the definitions of network coherence and constructions of web graphs. Calculations of network coherence are presented in Section 3. Section 4 includes the conclusions and discussions.

2. Network coherence and constructions of web graphs

2.1. Definitions

Network coherence characterizes the variance of fluctuations in first and second order consensus systems with the additive noise [15]. It has been shown that the network coherence could be determined by the spectra of Laplacian matrix [14,22,23]. Let the eigenvalues of Laplacian matrix be $0 = \lambda_1 < \lambda_2 \leqslant \ldots \leqslant \lambda_N$, then the first and second order network coherence are

$$H^{(1)} = \frac{1}{2N}\sum_{i=2}^{N}\frac{1}{\lambda_i} \tag{1}$$

and

$$H^{(2)} = \frac{1}{2N}\sum_{i=2}^{N}\frac{1}{\lambda_i^2}. \tag{2}$$

2.2. Web graphs

The web graphs $P_m(g)(g \geqslant 0)$ are formed by a positive-integer parameter $m(m \geqslant 3)$ after g generations. Initially at $g = 0,\ P_m(0)$ is a graph with $2m$ nodes and the edges of the outer cycle removed, see Fig. 1. For $g \geqslant 1,\ P_m(g)$ is obtained from $P_m(g-1)$, where the outer nodes in $P_m(g-1)$ form a new m-polygon and produce new nodes. Fig. 2 illustrate the processes at four generations with $m = 4$.

Based on the topology of web graphs, we obtain the total number of nodes N_g and edges E_g as $N_g = m(g+2)$ and $E_g = 2m(g+1)$.

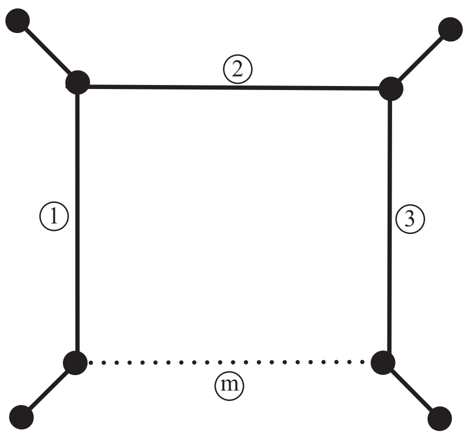

Fig. 1. The initial state $P_m(0)$ of web graphs.