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Nonsingular decoupled terminal sliding-mode control for a class of fourth-order nonlinear systems

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ABSTRACT

This paper presents a nonsingular decoupled terminal sliding mode control (NDTSMC) method for a class of fourth-order nonlinear systems. First, the nonlinear fourth-order system is decoupled into two second-order subsystems which are referred to as the primary and secondary subsystems. The sliding surface of each subsystem was designed by utilizing time-varying coefficients which are computed by linear functions derived from the input–output mapping of the one-dimensional fuzzy rule base. Then, the control target of the secondary subsystem was embedded to the primary subsystem by the help of an intermediate signal. Thereafter, a nonsingular terminal sliding mode control (NTSMC) method was utilized to make both subsystems converge to their equilibrium points in finite time. The simulation results on the inverted pendulum system are given to show the effectiveness of the proposed method. It is seen that the proposed method exhibits a considerable improvement in terms of a faster dynamic response and lower IAE and ITAE values as compared with the existing decoupled control methods.

1. Introduction

Sliding mode control (SMC) has received much attention due to its major advantages such as guaranteed stability, robustness against parameter variations, fast dynamic response and simplicity in implementation and therefore has been widely applied to control nonlinear systems [1–3]. The design of a SMC consists of two steps: design of a sliding surface and design of a control law. Once a suitable sliding surface and a suitable control law are designed, the system states can be forced to move towards the sliding surface and slide on the surface until the equilibrium (origin) point is reached. In most SMC schemes, the most commonly used sliding surface is the linear sliding surface which is based on linear combination of the system states by using an appropriate time-invariant coefficient. Despite the very well known advantages of SMC, it suffers from the time-invariant (constant) coefficient utilized in the linear sliding surface which slows the convergence rate of the system. Although the convergence rate can be made faster by adjusting this coefficient, again the system states in the sliding mode cannot converge to equilibrium point in finite time.

In order to achieve finite time convergence of the system states, a terminal sliding mode control (TSMC) approach has been firstly proposed by Zak [4]. Authors in [5–7] developed the first-order TSMC approach for the control of second-order nonlinear systems. In [8], the TSMC approach has been extended to high-order single-input and single-output linear systems. Zhihong et al. [9] developed a general TSMC for multi-input and multi-output linear systems. Although the existing TSMC approaches have the advantages of strong robustness against parameter variations and zero tracking error in finite time, they suffer from the singularity problem which generates an unbounded control signal due to negative fractional power existing

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in the nonlinear sliding surface function. Furthermore, the convergence speed is slowed down when initial values of the system states are far away from the equilibrium point. In [10], a fast terminal sliding mode control (FTSMC) method has been proposed to obtain a faster finite time convergence. Although the proposed FTSMC exhibits a high convergence rate for all initial values of the system states, the singularity problem still needs to be resolved. As a remedy to this problem, a nonsingular terminal sliding mode control (NTSMC) approach has been proposed [11]. Feng et al. [12] applied the NTSMC approach with decomposed control to multivariable linear systems. Recently, a continuous finite-time control approach has been proposed using a new form of terminal sliding modes [13]. It is shown that the proposed TSMC approach eliminates the singularity problem if the fractional powers are chosen properly and achieves a faster convergence compared with the conventional TSMC.

However, the TSMC, FTSMC and NTSMC approaches proposed in all of these studies are only suitable for second-order systems. The performance of these approaches for a class of fourth-order system is still questionable. The main reason of this comes from the fact that there are four states in the dynamic representation of such systems which should be controlled simultaneously by a single-input. The mechanical systems with fewer controls than the number of configuration variables to be controlled are known as underactuated systems. Underactuated systems have broad applications such as space robots, underwater robots, Pendubot (two-link robot), Acrobat, overhead crane and cart–pole. The Pendubot is generally used for research on nonlinear control [14]. The robust control strategy for Acrobat was proposed in [15]. The gain scheduling control approach was proposed for overhead cranes in [16]. A cart–pole (single-inverted pendulum) system holds some interesting challenges from nonlinear control point of view. For example, when a cart–pole system is controlled by the TSMC, FTSMC and NSTSMC approaches developed for a second-order nonlinear system, either pole or cart can be controlled successfully, but not both. This means that when some of the states reach equilibrium point in a finite time, the other states will not be controlled properly. A remedy to this problem is to decouple the system states and apply a suitable control law to stabilize the whole system. The first decoupled sliding mode control (DSMC) has been presented in [17]. Afterwards, a number of alternative decoupling methods have been proposed to overcome the decoupling issue [18,19]. Recently, a new decoupled sliding mode control method based on time-varying sliding surface slope (TVSSS) has been developed [20]. Despite the faster convergence rate obtained by this method, the system states still cannot converge to the equilibrium point in finite time.

In this paper, a nonsingular decoupled terminal sliding mode control (NDTSMC) is proposed for a class of fourth-order systems. The system under consideration was firstly decoupled into two second-order subsystems. Then a separate sliding surface function was defined for each subsystem in which the coefficients are time-varying. Afterwards, the proposed method has been applied to each subsystem separately to ensure that the states of both subsystems convergence to their equilibrium points in finite time. The main idea of NDTSMC is not only to decouple the system states, but also to ensure that the system states converge to the equilibrium in finite time. The proposed control method is applied to control a cart–pole system. Simulations are carried out and the results are compared with the existing methods such as TVSSS and DSMC.

The rest of this paper is organized as follows. Section 2 reviews the terminal sliding mode control for second-order systems. In Section 3, the proposed control strategy, time-varying coefficient computation with and without fuzzy logic, and stability analysis are described. In Section 4, the proposed method is used to control a cart–pole system and the simulation results are presented and compared with the existing decoupled methods. Finally, the conclusions are addressed in Section 5.

2. Second-order terminal sliding mode control

Consider a second-order nonlinear system represented by the following canonical state-space form

$$\dot{x}_1(t) = x_2(t) \tag{1}$$

$$\dot{x}_2(t) = f(\mathbf{x}, t) + b(\mathbf{x}, t)u(t) + d(t) \tag{2}$$

where $\dot{x}_1(t)$ and $\dot{x}_2(t)$ are the derivatives of $x_1(t)$ and $x_2(t)$, respectively, $\mathbf{x} = [x_1, x_2]^T$ is the state vector, $f(\mathbf{x}, t)$ and $b(\mathbf{x}, t) \neq 0$ are nonlinear functions representing system dynamics, $u(t)$ is the control input, and $d(t)$ represents the external disturbance. The conventional TSMC is described by the following nonlinear sliding surface function

$$S = \lambda x_1^{\gamma} + \dot{x}_1 \tag{3}$$

where $\lambda > 0$, and $0 < \left(\gamma = \frac{q}{p}\right) < 1$ where p and q are positive odd integers satisfying $p > q$. When the system is in the terminal sliding mode ($S = 0$), its dynamics can be determined by the following nonlinear differential equation

$$\dot{x}_1 = -\lambda x_1^{\gamma} \tag{4}$$

It has been shown [4] that $x_1 = 0$ is the terminal attractor of the system defined in (1) and (2). The equation in (4) can also be written as

$$dt = -\frac{dx_1}{\lambda x_1^{\gamma}} \tag{5}$$