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Bifurcation analysis of periodic orbits of a non-smooth Jeffcott rotor model

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ABSTRACT

We investigate complex dynamics occurring in a non-smooth model of a Jeffcott rotor with a bearing clearance. A bifurcation analysis of the rotor system is carried out by means of the software TC-HAT [25], a toolbox of AUTO 97 [6] allowing path-following and detection of bifurcations of periodic trajectories of non-smooth dynamical systems. The study reveals a rich variety of dynamics, which includes grazing-induced fold and period-doubling bifurcations, as well as hysteresis loops produced by a cusp singularity. Furthermore, an analytical expression predicting grazing incidences is derived.

1. Introduction

Rotating machines are very popular in engineering applications, ranging from optical disk drives and washing machines to turbojet engines and power generators. The study of their dynamics has attracted much attention, and special emphasis has been put on the development and refinement of mathematical models for understanding different phenomena observed in rotor systems. In general, the performance of rotating machinery can be enhanced by increasing the speed of rotation and decreasing the radial clearance between the rotating and non-rotating parts. This, however, significantly increases the risk of intermittent contacts between the components of rotor systems due to forced vibrations, resulting not only in possible costly and catastrophic mechanical failures (e.g. in aircraft jet engines) but also in a threat to the health of workers (e.g. hand-arm vibration syndrome [12]).

A common cause of vibration in rotating machinery is mass imbalance. This occurs when the principal axis of the moment of inertia of the rotating component does not coincide with the axis of rotation. Such eccentric rotors undergo periodic oscillation known as whirl. In practice, a rotor cannot be balanced perfectly, no matter what method is used, and the best achievable state of balance at the beginning of the operating life of a rotor tends to deteriorate with use. This fact has motivated many studies on rotor systems subjected to out-of-balance phenomena, see e.g. [10,11,15,17,21]. Most of the mathematical models used in these investigations are based on the Jeffcott rotor [13], which consists of a large unbalanced disk mounted midway between the bearing supports on a flexible shaft of negligible mass. Although the Jeffcott rotor model is an oversimplification of real rotors, it has proven to be very useful for understanding many important phenomena observed in real applications [7,9,26].

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Several investigations have revealed that the presence of bearing clearances in unbalanced rotor systems leads to complex dynamical behavior. This can be seen from the nonlinear nature of the mathematical models describing such systems, which are, in many cases, also non-smooth, giving rise to the possibility of nonconventional bifurcations such as grazing and sliding to occur. Ehrich [8] showed that unbalanced rotors operating eccentrically within a clearance behave as bilinear oscillators, where chaotic and subcritical superharmonic responses can be observed. Goldman and Muszynska [10] introduced general models to study the rotor–stator interaction under rubbing conditions (see also [20]). The transition from no contact to contact between the mechanical elements is described by means of variable stiffness, viscous damping and intermittent friction. A simpler horizontal Jeffcott rotor with bearing clearances is examined by Choi and Noah [2]. Their analysis focuses on the alternating periodic, aperiodic and chaotic responses, which are shown to be ruled by the Farey number tree. The classical bifurcation scenarios along with stability analysis of unbalanced rotors can be found in [2,3,14,17,18].

In the present work we consider a two-degrees-of-freedom rotor system with a bearing clearance, excited by an out-of-balance rotating mass, as shown in Fig. 2.1(a). As already mentioned in the previous paragraph, this type of physical configuration allows for non-smooth phenomena to occur, such as *discontinuity-induced bifurcations* [4]. However, little work has been reported on the analysis of such bifurcations in rotor systems. Recent results regarding grazing bifurcations include [3,22,23], where their approach is based on numerical time integration over large intervals. A rigorous study of grazing phenomena in rotor systems appears to be unavailable. In the present article, we will attempt to extend the current understanding of this phenomenon.

Another field of study that has received little attention is the bifurcation analysis of periodic orbits by means of path-following methods. As most of the mathematical models of rotors with clearances are piecewise smooth, the numerical continuation of periodic solutions requires the assembly of multiple boundary value problems, resulting in a continuation problem of large dimension. Investigations in this direction can be found in [24,27], where the authors apply reduction methods and frequency-domain techniques, respectively, for the implementation of the path-following algorithms. In the present work we will study numerically the discontinuity-induced bifurcations of periodic orbits of the two-degrees-of-freedom model shown in Fig. 2.1(a). For this purpose, we will use the software package TC-HAT [25], a continuation toolbox for the bifurcation analysis of periodic trajectories of non-smooth dynamical systems. It functions as a driver to a modified version of AUTO 97 [6].

The organization of this article is as follows. In Section 2, we describe in detail the physical configuration of the rotor system under consideration. The mathematical model [15,16] is appropriately adopted in order to perform the numerical analysis of the system by means of TC-HAT. This analysis is presented in Section 3. Here, we divide the study into two sets of parameters, the first one allowing grazing phenomena and the second one chaotic motion. In Section 4, we derive an analytical expression for the parameter values producing grazing bifurcations. This allows for a careful study of the different grazing scenarios when the parameters are varied. Finally, we present some conclusions and closing remarks concerning our study.

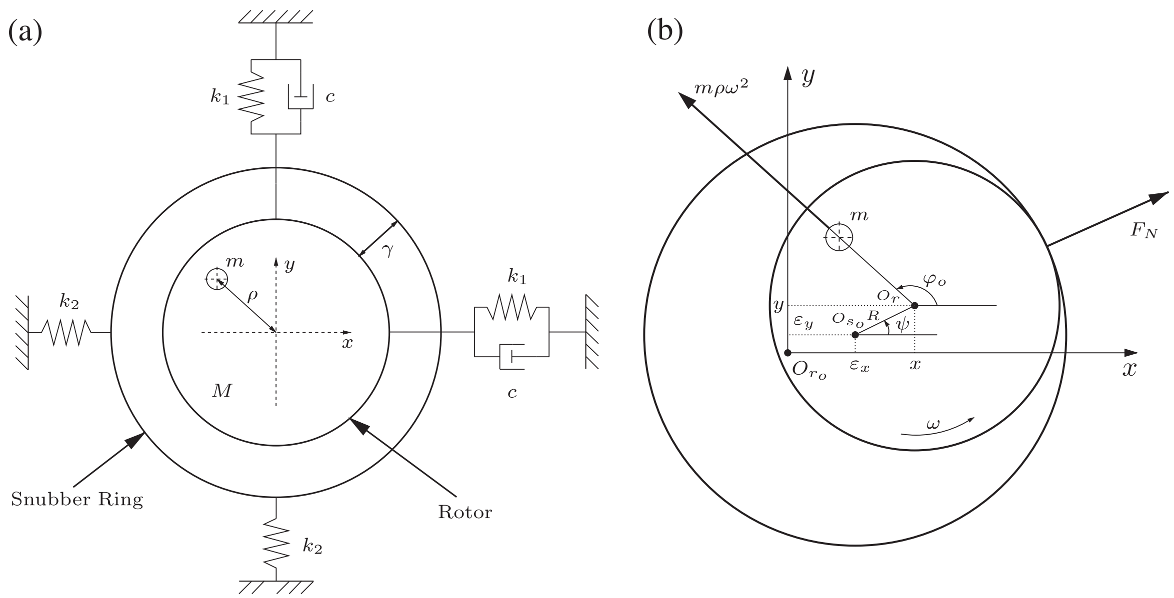

Fig. 2.1. (a) Physical model and (b) geometrical representation of the Jeffcott rotor with a snubber ring as proposed in [16]. The rotor of mass M is excited by an out-of-balance rotating mass producing a centrifugal force of magnitude $m\rho\omega^2$. When the rotor is contact with the snubber ring (i.e. when $R \geqslant \gamma$), a normal force F_N is generated, whose magnitude depends on the displacement of the snubber ring with respect to its equilibrium position O_{s_0}, see (2.3).