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New conditions for synchronization in complex networks with multiple time-varying delays ☆

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ABSTRACT

In this paper, two kinds of synchronization problems of complex dynamical networks with multiple time-varying delays are investigated, that is, the cases with fixed topology and with switching topology. For the former, different from the commonly used linear matrix inequality (LMI) method, we adopt the approach basing on the scramblingness property of the network's weighted adjacency matrix. The obtained result implies that the network will achieve exponential synchronization for appropriate communication delays if the network's weighted adjacency matrix is of scrambling property and the coupling strength is large enough. Note that, our synchronization condition is very new, which would be easy to check in comparison with those previously reported LMIs. Moreover, we extend the result to the case when the interaction topology is switching. The maximal allowable upper bounds of communication delays are obtained in each case. Numerical simulations are given to demonstrate the effectiveness of the theoretical results.

1. Introduction

The complex networks have received a major attention from various research communities. This is mainly due to their wide applications in many areas ranging from physics to biological, social and computer sciences. Synchronization, as a common phenomenon of a population of dynamically interacting units, is one of the most demonstrating topics in complex networks [1,2]. In fact, synchronization is a ubiquitous phenomenon in nature, synchronization phenomena has been found in different forms both in nature and in man-made systems, such as fireflies in the forest, applause, description of hearts, distributed computing systems, chaos-based communication network, and so on. Recently, many interesting results on synchronization were derived for various complex networks associated with models being time-varying coupling [3–5], switching phenomena [6–8], impulsive effects [9,10,17], and time delay [11–17].

In practical situations, time delays caused by signal transmission affect the behavior of coupled systems. Hence, various techniques have been used to deal with the synchronization problems of complex networks with coupling delays. In some cases, the linear matrix inequality (LMI) method is used to deal with such problems [11–17]. For example, in [11–14], the synchronization problems of complex networks with constant coupling delay were investigated. The synchronization conditions for both delay-independent and delay-dependent asymptotical stabilities in terms of LMIs were obtained. In [17], global synchronization of a linearly hybrid coupled network with time-varying delays was considered. Several effective sufficient conditions of global synchronization were attained based on the Lyapunov function and a LMI. Although very interesting from the theoretical viewpoint, the LMI method would bring some slack variables and the presence of too many slack variables increases computation burden and restricts applications of the synchronization conditions. Furthermore, the LMI method

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can hardly be extended to the multiple time delay case. Therefore, in this paper, another effective method which may avoid bringing slack variables will be used to discuss synchronization of complex dynamical networks with multiple time delays.

In the following, two kinds of synchronization problems of complex dynamical networks with multiple time-varying delays are considered, that is, the cases with fixed topology and with switching topology. For the former, we show that the stability theorem of synchronization is easily derived just by using the approach based on the scramblingness property of the network's weighted adjacency matrix. The approach is based on the work of Hajnal back to the 1950s [18]. Hajnal investigated the weak ergodicity of nonhomogeneous Markov chains and proposed scrambling matrix, which plays an important role in the convergence of products of stochastic matrices. Therefore, this method is widely used to study discrete time consensus problem in [19–21] and others. Similar method has also been used to study continuous time consensus problem in [22,23] and synchronization problem in [24,25]. However, to the best of our knowledge, few people extend this approach to synchronization problems of complex dynamical networks with time delay. Here, with the help of this approach, we will give a simple condition to ensure the exponential synchronization of the network. To this purpose, we extend the concept of Hajnal's scrambling property from stochastic matrices to nonnegative matrices with zero diagonal entries. The obtained result implies that the network will achieve exponential synchronization for appropriate time-varying delays if the network's weighted adjacency matrix is of scrambling property and the coupling strength is large enough. Moreover, the result is extended to networks with switching topologies. The maximal allowable upper bounds of communication delays are obtained in each case.

The paper is organized as follows. Section 2 contains the problem formulation, Section 3 is the main results. Some simulation results are presented in Section 4. The conclusion is given in Section 5.

2. Problem formulation

We denote a weighted digraph by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where the set of nodes $\mathcal{V} = \{v_1, \ldots, v_N\}$ and $N \geqslant 2$, the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, $\mathcal{A} = [a_{ij}]$ is a weighted adjacency matrix with nonnegative adjacency elements a_{ij}. A directed edge of $\mathcal{G}$ is denoted by $e_{ij} = (v_i, v_j) \in \mathcal{E}$, i.e., e_{ij} is a directed edge from v_i to v_j. The adjacency elements associated with the edges of the graph are positive, i.e., $e_{ij} \in \mathcal{E}$ if and only if $a_{ji} > 0$. Moreover, we assume $a_{ii} = 0$ for all $i \in \{1, \ldots, N\}$. The set of neighbors of node v_i is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}\}$. The Laplacian matrix $L(\mathcal{G}) = [l_{ij}]$ of digraph $\mathcal{G}$ is defined by $l_{ij} = \sum_{j=1, j\neq i}^{N} a_{ij}$ for $i = j$ and $l_{ij} = -a_{ij}$ for $j \neq i$.

Consider delayed complex dynamical network consisting of N identical nodes, in which each node is an n-dimensional dynamical subsystem. The entire network is described by

$$\frac{dx^i(t)}{dt} = \mathcal{B}x^i(t) + f(x^i(t), t) + \epsilon \sum_{v_j \in \mathcal{N}_i} a_{ij} \Gamma(x^j(t - \tau_{ij}(t)) - x^i(t - \tau_{ij}(t))), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $x^i(t) = [x_1^i(t), x_2^i(t), \ldots, x_n^i(t)]^T \in R^n$ is the state vector of the ith node, and $\mathcal{B} = [b_{ij}] \in R^{n\times n}$ is a constant matrix, $f : R^n \times R^+ \mapsto R^n$ is a continuously function, $\epsilon > 0$ is the coupling strength, $\Gamma = \mathrm{diag}[\gamma_1, \gamma_2, \ldots, \gamma_n]$ is the weighted inner coupling matrix which is positive definite. The piecewise continuous functions $\tau_{ij}(t), i,j \in \{1, \ldots, N\}$ are time-varying communication delays and satisfy $0 \leqslant \tau_{ij}(t) \leqslant \tau$ for some $\tau > 0$. Denote $x^i(\theta) = \varphi^i(\theta) \in C([-\tau, 0], R^n)$ $(i = 1, \ldots, N)$, where $C([-\tau, 0], R^n)$ is the set of continuous functions from $[-\tau, 0]$ to R^n. Our control goal is to let the network achieve synchronization, i.e., $\lim_{t\to\infty} |x_k^i(t) - x_k^j(t)| = 0$ for all $i,j \in \{1, \ldots, N\}$ and $k \in \{1, \ldots, n\}$.

The communication topology among the group of agents may change dynamically due to link failure or creation, for instance, because of the limited detection range of agents, existence of the obstacles. In order to describe the switching topologies, we define a piecewise constant and right-continuous switching signal $\sigma(t)$ (σ in short): $[0, \infty) \mapsto \mathcal{F}_0$, where $\mathcal{F}_0 \subset Z$ is a finite index set. Therefore, the collection of all possible interaction topologies $\{\mathcal{G}_s = (\mathcal{V}, \mathcal{E}_s, \mathcal{A}_s) : s \in \mathcal{F}_0\}$ is a finite set.

Regarding the switching interconnection topologies, the corresponding network can be expressed as follows

$$\frac{dx^i(t)}{dt} = \mathcal{B}x^i(t) + f(x^i(t), t) + \epsilon \sum_{p=1}^{m} \sum_{j=1}^{N} a_{\sigma,ij}^p \Gamma x^j(t - \tau_p(t)), \quad i = 1, 2, \ldots, N, \tag{2}$$

where $\tau_p(\cdot) \in \{\tau_{ij}(\cdot) : i,j = 1, 2, \ldots, N\}$ for $p = 1, \ldots, m$ with $m \leqslant N(N-1)$, and $a_{\sigma,ij}^p$ is defined as follows

$$a_{\sigma,ij}^p = \begin{cases} 0, & j \neq i,\ \tau_p(\cdot) \neq \tau_{ij}(\cdot), \\ a_{\sigma,ij}, & j \neq i,\ \tau_p(\cdot) = \tau_{ij}(\cdot), \\ -\sum\limits_{j\neq i} a_{\sigma,ij}^p, & j = i, \end{cases} \tag{3}$$

where $a_{\sigma,ij}$ is the (i,j)th entry of matrix $\mathcal{A}_\sigma$. It is easy to see that $\sum_{j=1}^{N} a_{\sigma,ij}^p = 0$ for all i and p. Therefore, $\mathcal{A}^p(\mathcal{G}_\sigma) = [a_{\sigma,ij}^p]$ is a matrix that has nonnegative off diagonal entries and zero row sum for each p. Further, we can observe that

$$\sum_{p=1}^{m} a_{\sigma,ij}^p = \begin{cases} a_{\sigma,ij}, & j \neq i, \\ -\sum\limits_{j\neq i} a_{\sigma,ij}, & j = i, \end{cases} \tag{4}$$

i.e., $\sum_{p=1}^{m} \mathcal{A}^p(\mathcal{G}_\sigma) = -L(\mathcal{G}_\sigma)$, where $L(\mathcal{G}_\sigma)$ is the corresponding Laplacian matrix associated with $\mathcal{G}_\sigma$.