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$L_2 - L_\infty$ filtering for Markovian jump systems with time-varying delays and partly unknown transition probabilities ☆

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ABSTRACT

This paper considers the $L_2 - L_\infty$ filtering problem for Markovian jump systems. The systems under consideration involve time-varying delays, disturbance signal and partly unknown transition probabilities. The aim of this paper is to design a filter, which is suitable for exactly known and partly unknown transition probabilities, such that the filtering error system is stochastically stable and a prescribed $L_2 - L_\infty$ disturbance attenuation level is guaranteed. By using the Lyapunov–Krasovskii functional, sufficient conditions are formulated in terms of linear matrix inequalities (LMIs). A numerical example is given to illustrate the effectiveness of the proposed main results. All these results are expected to be of use in the study of filter design for Markovian jump systems with partly unknown transition probabilities.

1. Introduction

Signal estimation, which is a fundamental problem in signal processing, has received significant attention in the past decades. Filter is one of the effective and convenient tools for signal analysis. It is well recognized that one of the most popular ways to deal with the signal estimation is traditional Kalman filtering. This approach is based on the assumption that the system is exactly known and its disturbances are stationary Gaussian noises with known statistics [1]. When the systems noise disturbances are assumed to be arbitrary signals with bounded energy, the celebrated Kalman filtering scheme is no longer applicable. To overcome such restriction, some alternative approaches were introduced. To the best of the authors' knowledge, the published results can be categorized into three distinct approaches depending on the filtering performance criteria. The first category deals with H_∞ filtering where the input signal is assumed to be energy-bounded and the main objective is to minimize the energy of the estimation error for the worst possible bounded energy disturbance; see, e.g. [2–8], and the references therein. The second category treats energy-to-peak filtering where the objective is to minimize the peak-value of the estimation error for all possible bounded energy disturbances. For more results on this topic, we refer readers to [8–13]. The third category treats peak-to-peak filtering where the objective is to minimize the induced L_∞ gain of the estimation error for the unknown energy disturbances [14–16].

On the other hand, considerable attention has been devoted to Markovian jump systems (MJSs) due to their extensive applications in mechanical systems, economics, systems with human operators, and other areas [17]. A MJS is a hybrid sys-

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tem with state vector that has two components $x(t)$ and $r(t)$. The first one is in general referred to as the state, and the second one is regarded as the mode. In its operation, the jump system will switch from one mode to another in a random way, based on a Markovian chain with finite state space. For the traditional MJSs with completely known transition probabilities, many analysis and synthesis results have been reported, see, e.g. [18–20]. In fact, it is very hard and expensive to obtain precisely all the transition probabilities even for a simple system. For this reason, much attention has been focused on more general MJSs with partly unknown transition probabilities [21–26]. More recently, some results of filtering problem for MJSs have been established in terms of the linear matrix inequality, e.g. [4,27–29]. However, the above-mentioned results are obtained based on the assumption of complete knowledge on transition probabilities. Thus it is more practical and challenging to design a filter for the underlying systems with partly known transition probabilities, which has motivated this paper.

This article deals with the problems of $L_2 - L_\infty$ filtering for MJSs with time-varying delays and partly unknown transition probabilities. By using the Lyapunov–Krasovskii functional, a new mode delay-dependent sufficient condition on stochastic asymptotic stability with the $L_2 - L_\infty$ performance is derived in terms of linear matrix inequalities (LMIs). Base on this, the existence condition of the desired filter which guarantees stochastic stability and an $L_2 - L_\infty$ performance of the corresponding filtering error system is presented. A numerical example is provided to show the effectiveness of the proposed results.

Notation. Throughout this paper, $\lambda_{\max}(Q)$ and $\lambda_{\min}(Q)$ denote, respectively, the maximal and minimal eigenvalue of matrix Q. $(\Omega, \mathcal{F}, \mathcal{P})$ is a probability space, Ω is the sample space, $\mathcal{F}$ is the $\sigma -$ algebra of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $E\{\cdot\}$ refers to the expectation operator with respect to some probability measure $\mathcal{P}$. We use diag$\{\cdot,\cdot,\cdot\}$ as a block-diagonal matrix. $A > 0$ (<0) means A is a symmetric positive (negative) definite matrix, A^{-1} denotes the inverse of matrix A and A^T the transpose, I is the unit matrix.

2. System description and definitions

Fix a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and consider the following stochastic system:

$$\begin{cases} \dot{x}(t) = A(r(t))x(t) + A_\tau(r(t))x(t - \tau(r(t), t)) + B(r(t))v(t) \\ y(t) = C(r(t))x(t) + C_\tau(r(t))x(t - \tau(r(t), t)) + D(r(t))v(t) \\ z(t) = H(r(t))x(t) \\ x(t) = \phi(t), \ t \in [-\tau, 0], \end{cases} \tag{1}$$

where $x(t) \in \Re^n$ is the state vector; $v(t)$ is the disturbance input which belongs to $L_2[0\ \infty)$; $y(t) \in \Re^p$ is the measured output; $z(t) \in \Re^q$ is the signal to be estimated; $\phi(t)$ is a compatible vector-valued initial function defined on $[-\tau, 0]$; $A(r(t)), A_\tau(r(t)), B(r(t)), C(r(t)), C_\tau(r(t)), D(r(t))$ and $H(r(t))$ are real constant matrices with appropriate dimensions. $\{r(t), t \geqslant 0\}$ is a continuous-time Markovian process with right continuous trajectories and taking values in a finite set $\mathcal{S} = \{1, 2, \ldots, N\}$ with transition probability matrix $\Pi = \{\pi_{ij}\}$ given by

$$Pr\{r_{t+h} = j | r_t = i\} = \begin{cases} \pi_{ij}\triangle + o(\triangle), & i \neq j, \\ 1 + \pi_{ii}\triangle + o(\triangle), & i = j, \end{cases}$$

where $\triangle > 0$ and $\lim_{h\to 0} \frac{o(\triangle)}{\triangle} = 0$; $\pi_{ij} \geqslant 0$ for $i \neq j$ is the transition rate from mode i at time t to mode j at time $t + h$ and $\pi_{ii} = -\sum_{j=1, j \neq i}^{N} \pi_{ij}$. In this paper, the transition rates of the jumping process are considered to be partly accessible, i.e., some elements in matrix Π are unknown. For instance, for system (1) with 3 operation modes, the probabilities matrix may be as:

$$\begin{bmatrix} \pi_{11} & ? & ? \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix}, \quad \begin{bmatrix} \pi_{11} & ? & ? \\ ? & \pi_{22} & ? \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix}.$$

where “?” represents the inaccessible elements. For notational clarity, $\forall i \in \mathcal{S}$ the set U^i denotes $U^i = U^i_{kn} \cup U^i_{uk}$ where $U^i_{kn} \triangleq \{j : \pi_{ij}$ is known for $i \in \mathcal{S}\}, U^i_{uk} \triangleq \{j : \pi_{ij}$ is unknown for $i \in \mathcal{S}\}$. In addition, we denote $S^i_{kn} \triangleq \sum_{j \in U^i_{kn}} \pi_{ij}$. For simplicity, for each possible $r_t = i, \ i \in \mathcal{S}$, a matrix $R(r(t))$ will be denoted by R_i, for example, $A(r(t))$ is denoted by A_i, $A_\tau(r(t))$ is denoted by $A_{\tau i}$, and so on. $\tau_i(t)$ denotes the mode-dependent time delays when the mode is in $r(t)$ and satisfies

$$0 \leqslant \tau_i(t) \leqslant \tau_i \leqslant \tau, \quad \dot{\tau}_i(t) \leqslant \mu_i < 1. \tag{2}$$

In this paper, the following full-order linear filter is proposed to estimate the signal $z(t)$:

$$\dot{\hat{x}}(t) = A_{F_i}\hat{x}(t) + B_{F_i}y(t), \quad \hat{x}(0) = 0 \tag{3}$$

$$\hat{z}(t) = C_{F_i}\hat{x}(t) \tag{4}$$

where $\hat{x}(t)$ is the filter state vector and $(A_{F_i} \quad B_{F_i} \quad C_{F_i})$ are appropriately dimensioned filter matrices to be determined.

Define the estimation error by $e(t) = z(t) - \hat{z}(t)$, we obtain the following filtering error system: