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Novel rogue waves in an inhomogenous nonlinear medium with external potentials

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ABSTRACT

Employing the similarity transformation connected with the standard constant coefficient nonlinear Schrödinger equation, we obtain the analytical rogue wave solutions to a generalized variable coefficient nonlinear Schrödinger equation with external potentials describing the pulse propagation in nonlinear media with transverse and longitudinal directions nonuniformly distributed. Based on the obtained solutions, abundant structures of rogue waves are constructed by selecting some special parameters. The main properties as well as the dynamic behaviors of these rogue waves are discussed by direct computer simulations.

1. Introduction

Nonlinear science is believed by many outstanding scientists to be a most important frontier for understanding nature [1,2]. In this domain, solitary wave theory plays a very important role and has found wide applications in various branches of physics (see [3–8]) and in various other disciplines (see [9,10]). Many nonlinear phenomena and dynamical processes, for example, propagation of optical pulses, waves in plasmas, electromagnetic waves, and self-focusing in laser pulses, can be well described by the nonlinear Schrödinger equation (NLSE) also known as the Gross–Pitaevskii equation (GPE) in the context of Bose–Einstein condensate (BEC). Meanwhile, it is a most useful approach to understand the physical mechanism of natural phenomena described by NLSEs through finding various types of solutions, especially the ones of the solitary wave of NLSEs. In recent decades, some efficient methods have been proposed to construct various analytic solutions of the NLSEs (see [11–20], and others). Among these methods, the similarity transformation technique, transformation parameters of which can be determined by a set of differential equations, has been applied productively to the variable coefficient NLSEs [19,20] and helps to produce selected solutions in analytical form important for a variety of applications. Recently, spatial or spatiotemporal solitary waves have been intensively studied due to their novel physics and potential applications [4,21–25]. Various fundamental solitons [4,21,22], similaritons [23], rogue waves [24,25] have been predicted theoretically and observed experimentally.

Having different names like freak waves, monster waves and so on, rogue waves are spontaneous nonlinear waves with amplitudes significantly larger (two, three, or more times higher) than the surrounding average wave crests [26,27]. For a quite long time, rogue waves are thought to be mysterious since they appear from nowhere and disappear without a trace

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[28]. Generally speaking, they develop due to the interaction of the nonlinearity and dispersion in the wave propagation. Besides, soliton collisions with energy exchange can lead to their creation [29] and soliton resonance can also excite them [26]. Nowadays, for understanding better their physical mechanisms, the study of rogue waves by the Darboux transformation [28,30,31], the similarity transformation and the numerical simulation [24,25,32] has been received increasing attention. In fact, rogue waves can be regarded as a special type of solitary waves and have drawn much attention in some fields of nonlinear science (see [33–38]). The basic rogue wave solution of the NLSE called the Peregrine soliton or Peregrine breather was first derived by Peregrine [30]. Solli et al. [34,35] first observed the rogue waves in optical fiber and found that they could be used to stimulate supercontinuum generation. Recently, Akhmediev et al. [31] presented explicit forms of the rational solutions to describe them by the deformed Darboux transformation. Yan [39] and Ma et al. [40] investigated the nonautonomous rogue waves in one-dimensional and three-dimensional generalized NLSEs with variable coefficients by the similarity transformation and direct ansatz. More recently, Dai et al. [41] discussed their propagation behaviors in a variable coefficient higher-order NLSE by a similarity transformation connected with the constant coefficient Hirota equation.

In this paper, we consider a generalized variable coefficient NLSE model with varying dispersion, nonlinearity, gain and external potentials as follows [42,43]

$$iQ_t + \frac{1}{2}\beta(t)Q_{xx} + G(t)|Q|^2Q - \left(2\alpha(t)x + \frac{1}{2}\Omega(t)x^2\right)Q = i\frac{\gamma(t)}{2}Q \tag{1}$$

where $Q \equiv Q(x,t)$ is the complex envelope of the physical field, x is the transverse variable, and t is the longitudinal variable. $\beta(t)$ and $G(t)$ are the dispersion and nonlinearity management parameters, respectively. $\alpha(t)$ and $\Omega(t)$ represent the linear and harmonic oscillator potentials, and $\gamma(t)$ is the gain ($\gamma > 0$) or the loss ($\gamma < 0$) coefficient. Eq. (1) implies a large class of physical systems. In the context of nonlinear optics, Eq. (1) describes the pulse propagation in nonlinear media with transverse and longitudinal directions nonuniformly distributed. While in the context of BEC, Eq. (1) is also known as the generalized GPE describing the dynamics of matter wave solitons, where t and x represent the time and spatial coordinate, respectively. Under some special parameters, many authors have studied Eq. (1) from different perspectives [44–46]. For example, Li et al. [44] constructed one- and two-solitons for Eq. (1) with $\beta(t) = 1, G(t) = \mu$ (constant), $\Omega(t) = \gamma(t) = 0$ by the Hirota method. Serkin et al. [45] investigated nonautonomous solitons of Eq. (1) with $\gamma(t) = 0$. Atre et al. [46] presented a large family of exact solitary wave solutions of Eq. (1) with $\beta(t) = 1, \alpha(t) = 0$. Recently, Serkin and Hasegawa et al. [42] also

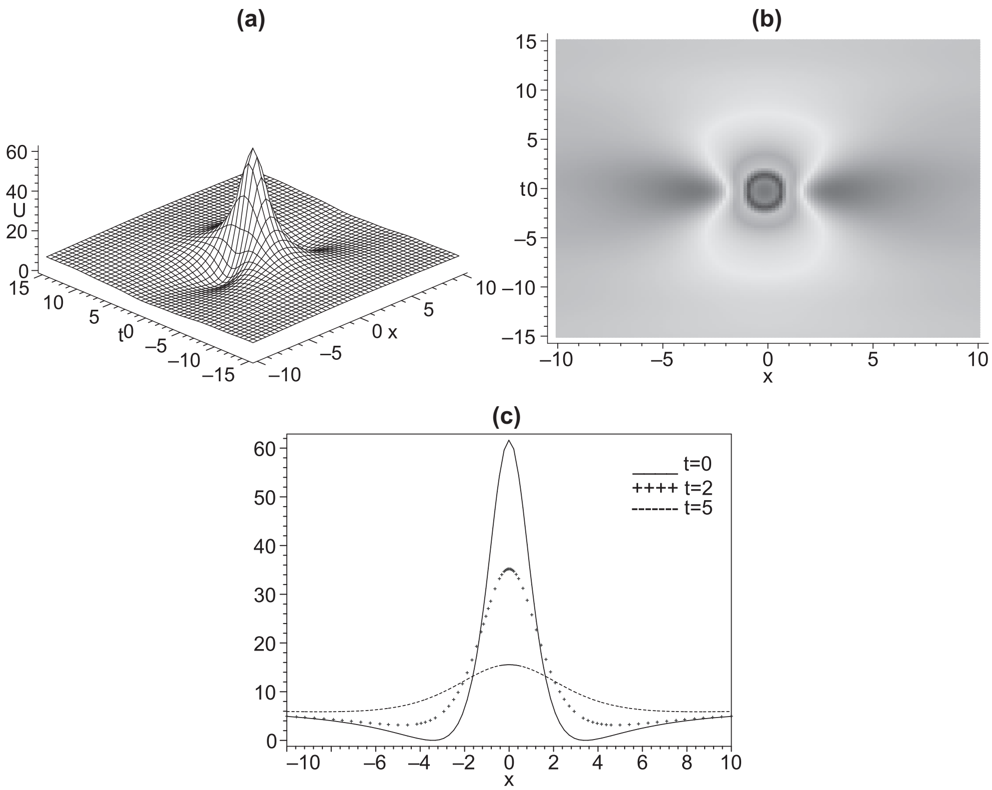

Fig. 1. Profiles of the one-rogue wave via solution (36) of Eq. (1) with parameters $\sigma = \delta = 1, C_1 = C_3 = C_5 = k = \alpha = 0, h = 1.2, \gamma = -0.001, C_2 = -0.5$, and $\beta = 1.35\exp(1 + 0.05\sin(t))$: (a) the intensity distribution $U_1 \equiv |Q_1|^2$; (b) the density distribution $|Q_1|^2$; (c) the intensity distribution $|Q_1|^2$ at different moments with $t = 0, t = 2$, and $t = 5$, respectively.