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Approximate controllability of nonlinear fractional dynamical systems

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ABSTRACT

In this paper, we consider the controllability problems for a class of nonlinear fractional differential equations of order $1 < q < 2$ with nonlocal conditions. In particular, a set of sufficient conditions are derived for the approximate controllability of nonlinear fractional dynamical systems by assuming the associated linear system is approximately controllable. Further, the result is extended to study the approximate controllability result for the nonlocal fractional control system with infinite delay. Also, as a remark, the conditions for the exact controllability results are obtained. The results are established by using solution operator theory, fractional calculations and fixed point techniques. Finally, an example is provided to illustrate the obtained theory.

1. Introduction

In the last three decades, fractional calculus has attracted many physicists, mathematicians and engineers, and notable contributions have been made to both the theory and applications of fractional differential equations (see [3] and references therein). Fractional order models of real systems are often more adequate than the usually used integer order models, since the description of some systems is more accurate when the fractional derivative is used. Also, fractional differential equations have recently proved to be valuable tools in modeling of many physical phenomena in various fields of science and engineering. Moreover, the qualitative behaviours such as the existence, controllability and stability of fractional dynamical systems are the current important issues dealt with by researchers. Existence of solutions to fractional differential equations involving the Riemann–Liouville fractional derivative or the Caputo fractional derivative have been paid much attention [28,35]. Also, in various real world problems, it is possible to require more measurements at some instances in addition to standard initial data and, therefore, the initial conditions changed to nonlocal conditions [2,12,39]. Since the nonlocal conditions give a better description in applications than standard initial data, Shu and Wang [36] studied the existence of mild solutions for fractional differential equations with nonlocal conditions of order $1 < \alpha < 2$.

Controllability is one of the important problem in mathematical control theory [24,25]. Since, the controllability notion has extensive industrial and biological applications [4,5]. In the literature, there are many different notions of controllability, both for linear and nonlinear dynamical systems [16,20,22,26,27]. Controllability of the deterministic and stochastic dynamical control systems in infinite dimensional spaces is well-developed using different kind of approaches (see [13,14,23,33]). It should be mentioned that the theory of controllability for nonlinear fractional dynamical systems is still in the initial stage. There are few works in controllability problems for different kind of systems described by fractional differential equations

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such as work done in [6,8,17,38]. Debbouche and Baleanu [7] established the exact null controllability result for a class of fractional evolution nonlocal integrodifferential control system in Banach space via the implicit evolution system. Klamka [18] discussed the minimum energy control problem for infinite-dimensional fractional-discrete time linear systems and derived a set of necessary and sufficient conditions for exact controllability. Assuming the corresponding linear system is controllable, Sakthivel et al. [32] studied the controllability for class of fractional neutral control systems governed by abstract nonlinear fractional neutral differential equations. Wang and Zhou [1] investigated the complete controllability of fractional evolution systems without involving the compactness of characteristic solution operators.

Moreover, in general in infinite dimensional spaces, the concept of exact controllability is usually too strong [22]. Therefore, the class of fractional dynamical systems must be treated by the weaker concept of controllability, namely approximate controllability. However, the approximate control theory for fractional equations is not yet sufficiently elaborated. More precisely, there are very few contributions regarding the approximate controllability results for fractional dynamical systems in the literature [19,30,31]. Sakthivel and Ren [34] obtained approximate controllability results for nonlinear fractional dynamical systems with state dependent delay by using Schauder's fixed point theorem. Yan [9] proved the approximate controllability of control systems governed by a class of partial neutral functional differential systems of fractional order with state-dependent delay in an abstract space. More recently, Kumar and Sukavanam [21] derived a new set of sufficient conditions for the approximate controllability of a class of semilinear delay control systems of fractional order. Motivated by the above consideration, the main aim of the present article is to investigate the approximate controllability result for a class of fractional nonlinear dynamical systems with nonlocal conditions of order $1 < q < 2$ of the form

$$D_t^q x(t) = Ax(t) + Bu(t) + f(t, x(t)) + \int_0^t a(t-s)g(s, x(s))ds, \quad t \in J = [0, b], \tag{1}$$
$$x(0) + m(x) = x_0 \in X, \quad x'(0) + n(x) = x_1 \in X,$$

where D_t^q is the Caputo fractional derivative of order $q; A : D(A) \subset X \to X$ is a sectorial operator of type (M, θ, q, μ) on a Banach space X with the norm $\|\cdot\|$; the control function $u(\cdot)$ is given in $L^2(J, U); U$ is a Banach space; B is a bounded linear operator from U into X; the nonlinear functions $f, g : J \times X \to X$ are continuous; $a : J \to X$ is an integrable function on J and $m, n : X \to X$ are continuous functions.

In the present work, we will introduce a suitable mild solutions and establish set of sufficient conditions for the approximate controllability of fractional dynamical systems (1) without and with infinite delay. More precisely, by using some constructive control function, we transfer the controllability problem for nonlinear dynamical systems into a fixed point problem in a function space. Further, as a remark, exact controllability of the considered systems are discussed. In particular, the results on controllability of nonlinear fractional dynamical systems are derived by assuming the corresponding linear system is controllable. Finally, an example is given to illustrate the developed theory.

2. Preliminaries

In this section, we shall recall some basic definitions and lemmas which will be used in this paper. Let $L(X)$ denote the Banach space of bounded linear operators from X into X. Let $C(J, X)$ denote the space of all continuous functions from J into X with the norm $\|x\| = \sup_{t \in J} \|x(t)\|$.

Definition 2.1 [29]. The Caputo derivative of order $q > 0$ for a function $f : [0, \infty) \to R$ can be written as

$$D_t^q f(t) = \frac{1}{\Gamma(n-q)} \int_0^t (t-s)^{n-q-1} f^{(n)}(s) ds = I^{n-q} f^{(n)}(t), \quad n-1 < q < n, \; n \in N.$$

The Laplace transform of the Caputo derivative of order $q > 0$ is given as

$$L\{D_t^q f(t) : \lambda\} = \lambda^q \hat{f}(\lambda) - \sum_{k=0}^{n-1} \lambda^{q-k-1} f^{(k)}(0), \quad n-1 < q < n.$$

Definition 2.2 [36]. Let $A : \mathcal{D} \subseteq X \to X$ be a closed and linear operator. A is said to be a sectorial operator of type (M, θ, q, μ) if there exists $\mu \in R, 0 < \theta < \pi/2, M > 0$, such that the q-resolvent of A exists outside the sector $\mu + S_\theta = \{\mu + \lambda^q : \lambda \in \mathbb{C}, |Arg(-\lambda^q)| < \theta\}$ and $\|\mathcal{R}(\lambda^q, A)\| \leqslant \frac{M}{|\lambda^q - \mu|}, \; \lambda^q \notin \mu + S_\theta$.

Further, if A is a sectorial operator of type (M, θ, q, μ), then it is not difficult to see that A is the infinitesimal generator of a q-resolvent family $\{T_q(t)\}_{t \geqslant 0}$ in a Banach space, where $T_q(t) = \frac{1}{2\pi i} \int_c e^{\lambda t} \mathcal{R}(\lambda^q, A) d\lambda$ [36].

Lemma 2.3 [36]. *Let A be a sectorial operator of type (M, θ, q, μ). If f satisfies a uniform Hölder condition with exponent $\beta \in (0, 1]$, then the unique solution of the Cauchy problem*

$$D_t^q x(t) = Ax(t) + f(t), \quad t \in J = [0, b], \; 1 < q < 2, \tag{2}$$
$$x(0) = x_0 \in X, \quad x'(0) = x_1 \in X$$