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Global analysis of boundary and interior crises in an elastic impact oscillator

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ABSTRACT

The crisis phenomena of a Duffing–Van der Pol oscillator with a one-side elastic constraint are studied by the composite cell coordinate system method in this paper. By computing the global properties such as attractors, basins of attraction and saddles, the vivid evolutionary process of two kinds of crises: boundary crisis and interior crisis are shown. The boundary crisis is resulted by the collision of a chaotic attractor and a periodic saddle on the basin boundary. It is observed that there are two types of interior crises. One is caused by the collision of a chaotic attractor and a chaotic saddle within the interior of basin of attraction. The other one occurs because a period attractor collides with a chaotic saddle within the interior of basin of attraction. The saddles of system play an important role in the crisis process. The results show that this method is an efficient tool to perform the global analysis of elastic impact oscillators.

1. Introduction

Generally speaking, there are three different types of crises [1,2] in smooth systems, namely boundary crisis, interior crisis, and attractor merging crisis. A boundary crisis occurs when a chaotic attractor collides with a periodic saddle or a chaotic saddle on its basin boundary, which is respectively called the regular boundary crisis and the chaotic boundary crisis [3]. An interior crisis results in a sudden change in size and shape of the attractor and occurs when a chaotic or nonchaotic attractor [4] collides with the chaotic saddle within its basin of attraction. When two or more chaotic attractors simultaneously collide with a periodic saddle or a chaotic saddle on the basin boundary, an attractor merging crisis occurs. Boundary and interior crises have been also called discontinuous bifurcations (catastrophic and explosive bifurcations [5,6]), which are defined according to the discontinuity of the locus in phase space of an attractor [7].

In recent years, crisis and discontinuous bifurcation phenomena in non-smooth systems have aroused the concern of researchers. Mason and Piiroinen [8,9] showed that the boundary crisis phenomenon can be observed in an impacting system. Ing et al. [10] analyzed the bifurcations of a linear impact oscillator with a one-sided elastic constraint near grazing and in order to understand the observed bifurcation scenarios a global analysis (boundary crisis) is shown. Thota and Dankowicz [11] studied the continuous and discontinuous grazing bifurcations in impacting oscillators. Luo et al. [12] studied the periodic motions and bifurcations of a two-degree-of-freedom plastic impact oscillator and found some discontinuous bifurcation phenomena. Rakshit et al. [13] analyzed the discontinuous bifurcation phenomena in the two-dimensional discontinuous maps by presenting a systematic approach. However, the global evolutionary process of crisis phenomena were not given in these works. In this paper, we will investigate the boundary and interior crises of an elastic impact oscil-

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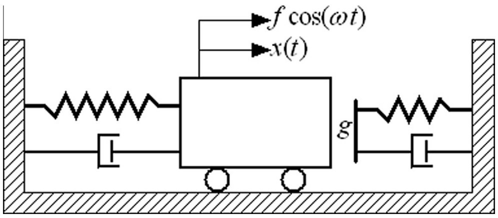

Fig. 1. Schematic of the elastic impact oscillator.

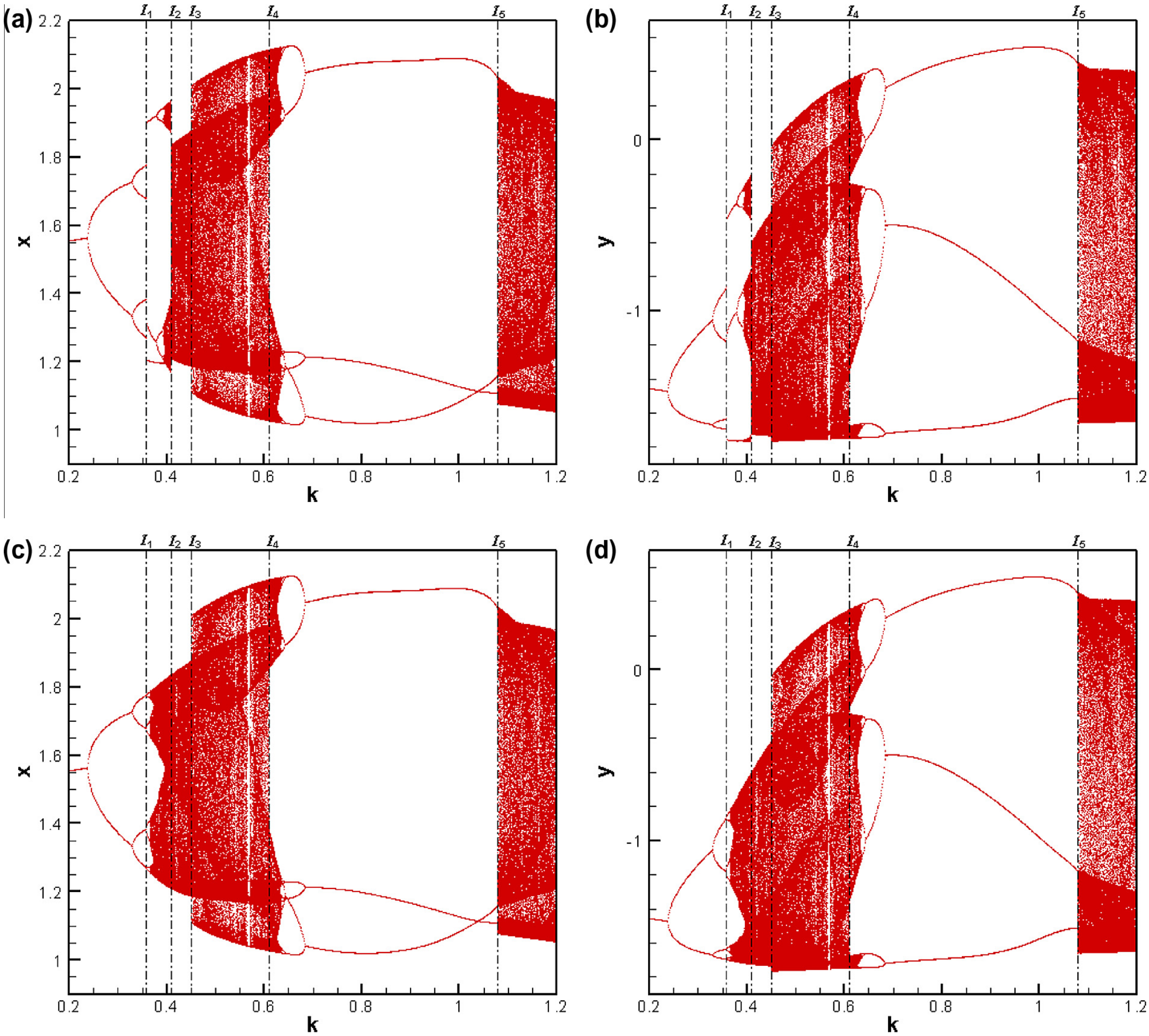

Fig. 2. The bifurcation diagrams of system (2) with different initial values. (a) and (b) $x = 1.2,\ y = -0.45$, (c) and (d) $x = 1.6,\ y = -1.5$.

lator from the global point of view. And the vivid evolutionary process of crisis will be shown. It will be found that the saddles play an important role in the crisis process.

The generalized cell mapping method [14,15] is an efficient approach to perform global analysis of dynamical systems. By introducing the digraph algorithms, Hong and Xu [16] presented the generalized cell mapping digraph method, which can obtain the attractors, basins of attraction, basin boundaries and unstable solutions of dynamical systems. By this method, they found some rich phenomena of crisis and bifurcation [17,18]. According to a new classification of transient cells,