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Numerical investigation of a three-dimensional four field model for collisionless magnetic reconnection

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ABSTRACT

In this paper we present the numerical investigation of a three-dimensional four field model for magnetic reconnection in collisionless regimes. The model describes the evolution of the magnetic flux and vorticity together with the perturbations of the parallel magnetic and velocity fields. We explored the different behavior of vorticity and current density structures in low and high β regimes, β being the ratio between the plasma and magnetic pressure. A detailed analysis of the velocity field advecting the relevant physical quantities is presented. We show that, as the reconnection process evolves, velocity layers develop and become more and more localized. The shear of these layers increases with time ending up with the occurrence of secondary instabilities of the Kelvin–Helmholtz type. We also show how the β parameter influences the different evolution of the current density structures, that preserve for longer time a laminar behavior at smaller β values. A qualitative explanation of the structures formation on the different z-sections is also presented.

1. Introduction

Magnetic reconnection is a fundamental process in highly conductive fluids and plasmas [1,2]. It can be defined as a change in the topology of the magnetic field lines, which decouple their motion from the fluid one. It is associated with a release of magnetic energy into heat, plasma kinetic energy and fast particle energy and it is characterized by the formation of current density sheets in the reconnection region along with strong velocity layers. The range of phenomena involving magnetic reconnection is very wide. It includes solar flares, geomagnetic substorms, interaction of the solar wind with the magnetopause, sawtooth oscillations and disruptions in laboratory plasma, such as in Tokamaks. One of the problems in magnetic reconnection is to identify the appropriate generalized Ohm's law and the physical mechanisms which cause the diffusion of the magnetic field through the plasma. In weakly collisional plasmas, such as the high temperature ones in Tokamaks, the inverse of the electron–ion collision frequency is larger than the relaxation time of internal sawtooth oscillations. This consideration made collisionless reconnection to become a frontier subject in the early 1990 [3,4]. In this regime the typical length scale of the reconnection process is given by the collisionless skin depth, d_e. Although a kinetic approach should be invoked in order to treat such collisionless regimes, in the presence of a strong guide field, fluid models, which offer a computational advantage, are often used. Many studies have been done in the framework of two-fluid models. In this context a description of the plasma behavior can be made considering a simpler two-field description [5] or a more

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sophisticated four-field description [6]. In particular, the two-field model in [5] has been extensively analyzed in the last decade both in two-dimensional and three-dimensional configurations [7–10]. In this model the evolution of the magnetic flux and plasma stream function is followed assuming that variations of the magnetic field and of the plasma velocity, along the direction parallel to the guide field, are negligible. The fingerprint of this approach is the coupling between the evolution of the current density and vorticity fields, which evolve in ordered or turbulent structures depending on the value of the electron temperature, which enters the equation through the ion sound Larmor radius $\varrho_{\mathbf{s}}$ [7,9], which is a further characteristic scale length of the phenomenon. For values of $\varrho_s \geqslant d_e$ the velocity layers, advecting the current density and vorticity, evolve in ordered coherent structures aligned with the separatrix of the magnetic island. On the other hand, for $\varrho_s \ll d_e$ the velocity layers tend to be aligned with the neutral line giving rise to strong shears that lead to the onset of secondary Kelvin–Helmholtz instabilities. This twofold picture has been also confirmed in three-dimensional configurations [10,11]. When we allow the system to develop also magnetic and plasma velocity perturbations along the direction parallel to the guide field, by considering the four-field model in [6], the picture becomes richer and a new scenario appears, where the evolutions of the current density and vorticity fields are no longer coupled. The crucial role in this change is played by the β parameter, expressing the ratio between the plasma and the magnetic field pressures. Indeed, when the low β limit is abandoned, the effects of the strong velocity shears that develop in the reconnection region, on one hand lead to a turbulent vorticity, but on the other hand they get suppressed in the evolution of the current density [11,12].

Recently, an extension of this four-field model to three dimensions has also been derived [13]. The model belongs to the class of fluid models for plasma physics, for which a noncanonical Hamiltonian formulation is known. The origin of this class can be traced back to the seminal work of Morrison and Greene [14] on ideal magnetohydrodynamics. The class was subsequently enlarged to a great extent by Phil Morrison and co-workers over the years, with the discovery of Hamiltonian structures for several reduced fluid models.

In this article we analyze this model by performing numerical simulations with the aim of investigating the evolution, and its dependence on β, of velocity and current density fields in a full 3D setting. In particular we address the question of understanding in what fields the secondary Kelvin–Helmholtz instability dominates and in what fields it is suppressed, and how such instability extends along the z direction. We also recall that the absence of an ignorable coordinate raises the problem of interpreting a dynamics which is much less constrained than the two-dimensional one[15,10,11]. In particular, the system no longer possesses the infinity of Casimir invariants which constrain the 2D case and in terms of which it is possible to explain the different structures observed in the current density and vorticity fields [5,8,16,12]. Such explanation was indeed possible, due to the presence of advected scalar fields whose existence was related to the presence of an infinite number of Casimirs. The paper is organized as follows: in Section 2 we present the model equations; in Section 3 the simulation results are shown and discussed; in Section 4 we focus on the energy partition; conclusions are drawn in Section 5.

2. Model equations

Our investigation is based on the 3D four-field model for collisionless reconnection, described in Ref. [13]. Considering a Cartesian coordinate system (x,y,z), with the constant magnetic guide field directed along z, the model equations are given by

$$\frac{\partial\left(\psi - d_e^2\nabla_\perp^2\psi\right)}{\partial t} + \left[\varphi, \psi - d_e^2\nabla_\perp^2\psi\right] - d_\beta[\psi, Z] + \frac{\partial\varphi}{\partial z} + d_\beta\frac{\partial Z}{\partial z} = 0, \tag{1}$$

$$\frac{\partial Z}{\partial t} + [\varphi, Z] - c_\beta[v, \psi] - d_\beta\left[\nabla_\perp^2\psi, \psi\right] - c_\beta\frac{\partial v}{\partial z} - d_\beta\frac{\partial\nabla_\perp^2\psi}{\partial z} = 0, \tag{2}$$

$$\frac{\partial\nabla_\perp^2\varphi}{\partial t} + [\varphi, \nabla_\perp^2\varphi] + \left[\nabla_\perp^2\psi, \psi\right] + \frac{\partial\nabla_\perp^2\psi}{\partial z} = 0, \tag{3}$$

$$\frac{\partial v}{\partial t} + [\varphi, v] - c_\beta[Z, \psi] - c_\beta\frac{\partial Z}{\partial z} = 0, \tag{4}$$

where ψ is the poloidal magnetic flux function, φ is the $\mathbf{E}\times\mathbf{B}$ stream function, Z is proportional to the parallel magnetic perturbation and v is the parallel plasma velocity. The parameter d_e represents the electron skin depth, whereas the other two parameters c_β and d_β are defined by $c_\beta \equiv \sqrt{\beta/(1+\beta)}$, with β indicating the ratio between the plasma pressure and the toroidal magnetic pressure, and by $d_\beta \equiv d_i c_\beta$, where d_i is the ion skin depth. The Poisson bracket is defined by $[f,g] = \hat{z}\cdot(\nabla f \times \nabla g)$ for generic fields f and g, whereas the symbol $\nabla_\perp$ refers to the gradient perpendicular to $\hat{z}$. Eqs. (1)–(4) are written in a dimensionless form, according to which, the time is normalized with respect to a characteristic Alfvèn time t_A, lengths are normalized with respect to a characteristic scale L and magnetic fields with respect to a characteristic value B. If such value is taken to be the amplitude B_0 of the toroidal guide field, then the following ordering applies:

$$\psi \sim \varphi \sim Z \sim v \sim \frac{\partial}{\partial t} \sim \frac{\partial}{\partial z} \sim \epsilon \equiv \frac{B_p}{B_0} \ll 1, \tag{5}$$