Commun Nonlinear Sci Numer Simulat 17 (2012) 2200–2214

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Modification of particle distributions by MHD instabilities I

R.B. White *

Plasma Physics Laboratory, Princeton University, P.O. Box 451, Princeton, NJ 08543, United States

ARTICLE INFO

Article history:
Available online 19 February 2011

Keywords:
Instabilities
Plasmas
Transport
Stochasticity
Beams
Alphas

ABSTRACT

The modification of particle distributions by low amplitude magnetohydrodynamic modes is an important topic for magnetically confined plasmas. Low amplitude modes are known to be capable of producing significant modification of injected neutral beam profiles, and the same can be expected in burning plasmas for the alpha particle distributions. Flattening of a distribution due to phase mixing in an island or due to portions of phase space becoming stochastic is a process extremely rapid on the time scale of an experiment but still very long compared to the time scale of guiding center simulations. Thus it is very valuable to be able to locate significant resonances and to predict the final particle distribution produced by a given spectrum of magnetohydrodynamic modes. In this paper we introduce a new method of determining domains of phase space in which good surfaces do not exist and use this method for quickly determining the final state of the particle distribution without carrying out the full time evolution leading to it.

1. Introduction

The resonant interaction of magnetohydrodynamic (MHD) modes and particle distributions can produce significant modification of the distribution and even induce large scale particle loss through profile avalanche, and is an important topic for magnetically confined plasmas. Low amplitude modes are known to be capable of producing significant modification of injected neutral beam profiles [1–5], and the same can be expected in burning plasmas for the alpha particle distributions. Since magnetic field ripple is a strong function of position, increasing rapidly near the plasma edge, a broadened profile can lead to an increase of stochastic trapped particle ripple loss [6]. Portions of phase space becoming stochastic lead to modification of the particle distribution, a process extremely rapid on the time scale of an experiment but still very long compared to the time scale of guiding center simulations, typically hundreds of hours of computing time to find saturated profiles under the action of a particular mode spectrum. In a typical experiment particles complete a toroidal transit in a microsecond, but bounce time in a small amplitude MHD resonance is about a millisecond, and tens of bounces are necessary for profile modification, so there are many orders of magnitude separation of the time scales involved. Previous work has focused on quasilinear models for the induced particle transport [7,8]. The subject of this paper is a method for determining the location and extent of mode-particle resonances and the final state of the particle distribution without carrying out the full time evolution leading to it. In Section 2 we discuss methods of determining the location and breadth of resonances, and introduce a new technique for determining the existence of good Kolmogorov Arnold Moser [9] (KAM) surfaces. In Section 3 we discuss transport induced by the presence of multiple incommensurate modes. In Section 4 we illustrate the determination of resonant domains with the new technique using some examples employing equilibria and mode spectra observed in DIII-D [2,3]. In Section 5 we examine a case of overlapping resonances producing a distribution avalanche, and in Section 6

* Tel.: +1 609 243 2628.
E-mail address: rwhite@pppl.gov

1007-5704/$ - see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.cnsns.2011.02.013

we construct a process of stochastic annealing, leading to the final state on a time scale large compared to the stochastic diffusion rate. In Section 7 are the conclusions.

2. Resonance determination

Using the guiding center drift approximation a particle orbit in an axisymmetric system is completely described by the values of the toroidal canonical momentum P_ζ, the energy E and the magnetic moment μ. Particle spatial coordinates are given by ψ_p, θ, ζ, respectively the poloidal flux coordinate, and the poloidal and toroidal angles. The magnetic field is given by

$$\vec{B} = g\nabla\zeta + I\nabla\theta + \delta\nabla\psi_p, \tag{1}$$

and in an axisymmetric equilibrium using Boozer coordinates g and I are functions of ψ_p only. The trajectory of the particle motion in the poloidal plane and the toroidal precession of the orbit are independent of the function δ [10,11].

The guiding center Hamiltonian is

$$H = \rho_\parallel^2 B^2/2 + \mu B + \Phi, \tag{2}$$

where $\rho_\parallel = v_\parallel/B$ is the normalized parallel velocity, $v_\parallel$ is the particle velocity parallel to the magnetic field, μ is the magnetic moment, and Φ the electric potential. The field magnitude B and the potential may be functions of ψ_p, θ and also ζ if axisymmetry is broken. Canonical momenta are

$$P_\zeta = g\rho_\parallel - \psi_p, \quad P_\theta = \psi + \rho_\parallel I, \tag{3}$$

where ψ is the toroidal flux, with $d\psi/d\psi_p = q(\psi_p)$, the field line helicity. Since we are interested in particle distributions, density is a relevant quantity. Volume is given by $dV = 2\pi J d\theta d\psi_p$, with the Jacobian $J = (gq + I)/B^2$ in Boozer coordinates. Magnetic surfaces of an equilibrium can be labeled by ψ_p, ψ, or V.

The equations of motion in Hamiltonian form are

$$\begin{aligned} \dot{\theta} &= \frac{\partial H}{\partial P_\theta} \quad \dot{P}_\theta = -\frac{\partial H}{\partial \theta}, \\ \dot{\zeta} &= \frac{\partial H}{\partial P_\zeta} \quad \dot{P}_\zeta = -\frac{\partial H}{\partial \zeta}. \end{aligned} \tag{4}$$

Equations for advancing particle positions in time, also in the presence of flute-like perturbations of the form $\delta\vec{B} = \nabla \times \alpha\vec{B}$ with $\vec{B}$ the equilibrium field and $\alpha = \sum_{m,n}\alpha_{m,n}(\psi_p)sin(n\zeta - m\theta - \omega_n t)$ can easily be derived [11]. Including such a perturbation the Hamiltonian for guiding center motion becomes $H = (\rho_c - \alpha)^2B^2/2 + \mu B + \Phi$, with $\rho_c = \rho_\parallel + \alpha$, and variables ζ, θ, ρ_c, ψ_p. In addition, for ideal MHD perturbations the rapid mobility of the electrons makes the electric field experienced by the ions parallel to the magnetic field equal to zero. With this form of the field perturbation it is necessary to add an electric potential Φ to cancel the parallel electric field induced by $d\vec{B}/dt$, with

$$\sum_{m,n} \omega B \alpha_{m,n} e^{i(n\zeta - m\theta - \omega t)} - \vec{B}\cdot\nabla\Phi/B = 0, \tag{5}$$

where we have neglected terms of order α^2. In Boozer coordinates, used in our simulations, taking $\Phi = \sum_{m,n}\Phi_{m,n}e^{i(n\zeta - m\theta - \omega t)}$ the solution is

$$(gq + I)\omega\alpha_{m,n} = (nq - m)\Phi_{m,n}, \tag{6}$$

but in general coordinates where $I = I(\psi, \theta)$ the solution is complicated by the coupling of different poloidal harmonics. The guiding center equations including MHD perturbations are realized using a fourth order Runge–Kutta algorithm in the code ORBIT [12]. The units are conveniently defined by the on-axis gyro frequency ω_0 (time) and the major radius R (distance).

The magnetic moment μ is conserved by the interaction of a particle with a mode with frequency much smaller than the cyclotron frequency, so only P_ζ and E are modified by interaction with it. For a given equilibrium and a fixed value of μ the domains of confined particles in the P_ζ, E plane are given by parabolas defining orbits that make contact with the magnetic axis, the low field side outer boundary, and the high field side outer boundary as well as the trapped-passing boundary.

An example is shown in Fig. 1 for a reversed shear equilibrium with $q = 4.7$ on axis and $q = 9$ at the plasma boundary. The major radius is $R = 168$ cm, minor radius $a = 65$ cm, and the field on axis is 20 kG. Shape and size of the various domains changes with the equilibrium parameters, but the general topology is always similar to that shown. The plane of P_ζ, E is shown for $\mu B_0 = 20$ keV with B_0 the magnetic field on axis and the particle distribution is limited to have energy between 20 and 60 keV. The apex of the parabolas are at $E = \mu\ B_{max}$ (label a) for the high field side (left edge, parabola with label L), $E = \mu B_0$ (label b) for the magnetic axis (parabola with label A), and $E = \mu B_{min}$ (label c) for the low field side (right edge, parabola with label R). The confined counter passing and co-passing orbits share a common triangular region, in which they have the same values of P_ζ and E but opposite signs of pitch. The small eye shaped region near point b consists of potato orbits, particles for which $v_\parallel$ vanishes along the orbit, but which circle the magnetic axis due to drift. The trapped particle domain borders both the co-passing and the counter passing domains along its upper boundary.