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Research paper

Snaking bifurcations in a self-excited oscillator chain with cyclic symmetry

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ABSTRACT

Snaking bifurcations in a chain of mechanical oscillators are studied. The individual oscillators are weakly nonlinear and subject to self-excitation and subcritical Hopf-bifurcations with some parameter ranges yielding bistability. When the oscillators are coupled to their neighbours, snaking bifurcations result, corresponding to localised vibration states. The snaking patterns do seem to be more complex than in previously studied continuous systems, comprising a plethora of isolated branches and also a large number of similar but not identical states, originating from the weak coupling of the phases of the individual oscillators.

1. Introduction

Spatially localised states of dynamical systems have been studied in a large number of different fields in the sciences and in engineering. While for linear systems Anderson localisation was the key to quite a satisfactory understanding, in nonlinear dynamical systems the quest to understand localisation seems far from settled. For a long time progress seems to have been largely confined to conservative nonlinear systems, where solitons and breathers made their appearance. Only later, dissipative systems have come into focus, with first work based on tracing solitons into the driven and dissipative regime, introducing dissipative solitons. In parallel to the study of solitary states in conservative and dissipative systems, another breakthrough to the understanding of spatial localisation in dissipative localisation was accomplished in the study of subcritical bifurcations in pattern-forming systems, where the concept of branching has emerged and is a well-established field of study today.

Branching is today well known in a number of disciplines, amongst others in optics [1], granular matter [2], structural mechanics [3–7], and mostly in fluid dynamics [1,8–12], and magnetohydrodynamics [13,14]. The first studies into the topic have probably emerged in the field of binary-fluid convection, where spatially localised convection rolls have been observed in water-ethanol mixtures [8] or helium [9]. There localised convection domains of arbitrary length are found to be stable, being surrounded by the conductive state.

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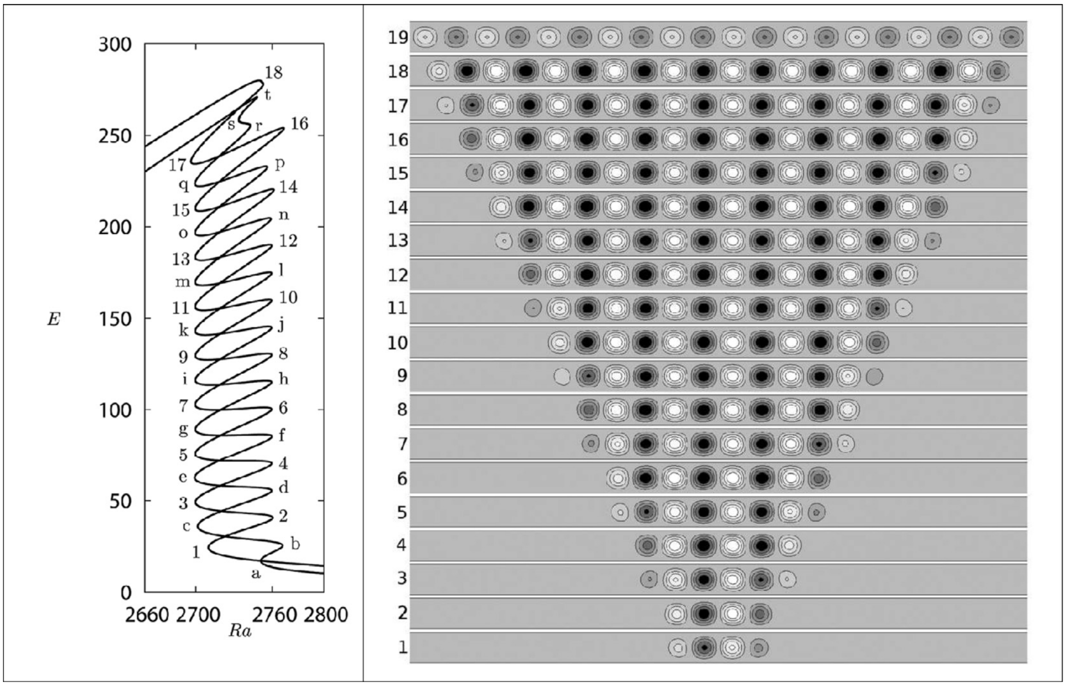

Fig. 1. An example for snaking in a convection system. Adapted from [11]. Left: typical snaking pattern with two intertwined solution branches in the bifurcation diagram with kinetic energy “E” of the fluid plotted versus the Rayleigh number “Ra”. Right: the stream-functions for solutions as marked in the snaking pattern. The graphs show the spatially localised convection patches.

In terms of bifurcation diagrams, the localised states have shown to be arranged in a unique and fascinating way, giving birth to what is called a snaking structure [6,10,11,14]. The snaking structure has e.g. been studied a lot in the one dimensional [15] and the two dimensional [16] Swift–Hohenberg equation, which is a convenient and generic model system to study fundamental properties of the arising dynamics.

A typical snaking bifurcation diagram involves two snaking solution branches, intertwined into each other. Fig. 1 gives an example from convection [11] to illustrate the phenomenon. For the bifurcation diagram (left panel), the average kinetic energy “E” of the flow is plotted versus the Rayleigh number “Ra” and two intertwined branches appear. In the right panel, nineteen solutions are shown, which correspond to the numbers positioned close to the snaking structure, in which spatially localised convective rolls can be identified. Notice that the higher the energy of the solution, the larger the number of convection cells. Often the two snaking branches are also interconnected through a number of unstable branches, and a ladder like pattern emerges [16].

Although snaking bifurcations are now generally known and studied in many fields of dynamical systems, it seems that there is hardly any study into the phenomenon in the context of structural vibrations in engineering. In many respects this is quite surprising, since non-linear oscillators with subcritical Hopf bifurcations, often coupled to neighbouring oscillators of the same type into chains or arrays, are actually very common models for a number of systems from engineering vibrations. And also the appearance of bi- or multi-stability, which is obviously at the core of the phenomenon [12,15–18], is well established in many of these engineering systems. Moreover, the emergence of spatially localised vibration states in structural dynamics is also a well known observational fact: e.g. in turbo-machinery, there is the so-called effect of ’mis-tuning in rotors’[19,20]. Traditionally, the origin of the localisation is thought to have its root in slight system inhomogeneities, leading to linear localisation in the sense of Anderson. From testing, strong localisation is confirmed, but proper validation of the theory has up to now not been accomplished in the linear framework. In a sense it is tempting to hypothesise that one of the key reasons behind might be the non-linearity involved, which definitely becomes substantial for the large local vibration amplitudes observed. To the best of our knowledge, in model systems for turbo-machinery dynamics, snaking behaviour has never been investigated. Also systems from fluid-structure-interaction, may show weak non-linearity, Hopf bifurcation, and bi-stability, like models for aerofoil flap dynamics [21–24]. Similarly in friction induced vibrations the emergence of snaking could be well expected, with all the necessary ingredients like flutter instability and bi-stability already known to exist, cf. e.g. [25,26].

We will thus consider a model system as simple as we can think of, but derived from models actually in use in the turbo-machinery community and the field of fluid-structure interaction and friction-induced vibration. We choose a chain of (weakly non-linear) oscillators coupled into a linear oscillator chain. For simplicity we close the chain into a cyclically symmetric ring, which moreover has the advantage of bringing it even closer to models used widely in turbo-machinery