Commun Nonlinear Sci Numer Simulat 44 (2017) 184–192

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

The transmissibility of nonlinear energy sink based on nonlinear output frequency-response functions

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ARTICLE INFO

Article history:
Received 30 April 2015
Revised 5 February 2016
Accepted 11 August 2016
Available online 12 August 2016

Keywords:
Nonlinear energy sink
Transmissibility
Frequency-response function
Cubic nonlinear stiffness

ABSTRACT

For the first time, a new representation of transmissibility based on nonlinear output frequency-response functions (NOFRFs) is proposed in the present study. Furthermore, the transmissibility is applied to evaluate the vibration isolation performance of a nonlinear energy sink (NES) in frequency domain. A two-degree-of-freedom (2-DOF) structure with the NES attached system is adopted. Numerical simulations have been performed for the 2-DOF structure. Moreover, the effects of NES parameters on the transmissibility of the nonlinear system are evaluated. By increasing the viscous damping and mass, as well as decreasing the cubic nonlinear stiffness of the NES, the analytical results show that the transmissibility of the 2-DOF structure with NES is reduced over all resonance regions. Therefore, the present paper produces a novel method for NES design in frequency domain.

1. Introduction

Vibration isolation is very important in many engineering practices, such as vibration control of satellite systems and sensitive equipment [1–3]. Generally speaking, there are two kinds of vibration isolation systems: passive and active. Active control strategies were shown to be effective in suppressing oscillations. However, they often associate with problems like high cost and added weight. Also, active control usually require an independent energy supply. These limitations highly restrict its usage in many applications. Therefore, passive vibration control is still used more often these days.

Linear absorber is commonly used to attenuate unwanted vibration energy in engineering practices but it is only effective in the vicinity of its design frequency [4]. On the other hand, the excitation frequency in reality is not fixed in most cases. Therefore, the linear vibration absorber will not be able to work efficiently in these situations due to its narrowband effectiveness. Even worse, the linear absorber may also adversely affect the frequency response in the neighbourhood of resonant frequencies of the primary system [5].

To overcome the limitations of the linear absorber, nonlinear strategies in vibration suppression have been widely studied over these years. In Refs. [6–9], it has been shown that energy from transiently loaded linear subsystems may be passively absorbed by properly designed, essentially nonlinear local attachments known as nonlinear energy sink (NES). A valuable feature of a NES is its capability to realize targeted energy transfer (TET). Therefore, passive energy pumped from the pri-

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http://dx.doi.org/10.1016/j.cnsns.2016.08.008
1007-5704/

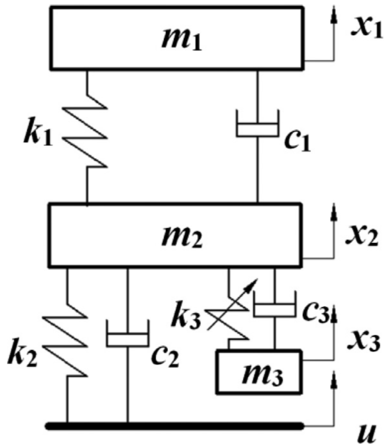

Fig. 1. A 2-DOF oscillator with a nonlinear energy sink attached.

mary system will be transferred to the NES irreversibly. Furthermore, these works also showed that a properly designed NES could dissipate vibrational energy within a wide frequency range as long as the input energy is above a certain threshold.

Over the last few years, much attentions have been paid to researches on NES. The effect of a NES on the steady-state dynamics of a weakly coupled system has been investigated theoretically and experimentally [10]. Many novel NES designs with different types of nonlinear stiffness, like cubic, non-smooth, non-polynomial, etc, have been proposed [11–15]. Capturing and storing the dissipated energy by the NES with a vibration energy harvester have been researched [16]. For better vibration isolation performance, some novel designs of NES, such as asymmetric NES and NES with negative linear and nonlinear coupling stiffness components, have also been proposed [17–18]. However, the above mentioned works regarding to NES analysis were conducted by evaluating displacement or energy dissipation in time-domain. Other works regarding to NES analysis in frequency domain were carried out by evaluating the frequency response through FFT [19–24]. For a linear system, transmissibility is usually represented as the ratio between the magnitude of the system output and input spectrum. Due to its simplicity, this representation is also widely used in industry for nonlinear frequency domain analysis and design. But, it should be noted that this representation for transmissibility is not correct theoretically for nonlinear systems [25].

Nonlinear output frequency response functions (NOFRFs) is a new concept proposed by Lang and Billings recently [25]. It has been used to detect the position of nonlinear components in periodic structures which is of great practical significance in system identification [26–27]. The concept of NOFRFs is regarded as an extension of the classical frequency response function for linear systems to the nonlinear case. By introducing the concept of NOFRFs, the analysis of nonlinear systems could be implemented in a way similar to the analysis of linear systems. Therefore, informative and physically meaningful interpretation for many nonlinear phenomena in the frequency domain are provided. In contrast to the representation of transmissibility based on FFT, the NOFRFs are well defined in the context of nonlinear systems. Therefore, the transmissibility results achieved based on NOFRFs should be more accurate in frequency domain analysis.

In the present study, a new representation of the transmissibility based on NOFRFs is proposed for the first time. Furthermore, the transmissibility is applied to evaluate the vibration isolation performance of a NES in frequency domain. A two-degree-of-freedom (2-DOF) structure with the NES attached system is adopted. Numerical simulations have been performed for the 2-DOF structure. Moreover, the effects of NES parameters on the transmissibility of the nonlinear system based on the concept of NOFRFs are evaluated. The analytical results show that the transmissibility is reduced over all resonance regions by increasing the viscous damping and mass, as well as decreasing the cubic nonlinear stiffness of the NES. These results are very useful for NES design in engineering practices.

2. Two degree of freedom system with NES

The systems considered in the present study are described by a typical 2-DOF oscillator as shown in Fig. 1 with the input $u(t)$ applied at the supporting base and a nonlinear energy sink attached to the 2th mass. The nonlinear oscillator is designated as the nonlinear energy sink (NES), and possesses a cubic nonlinear stiffness in parallel to a viscous damper.

The governing equations of motion of this system are given by

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} c_1 & -c_1 & 0 \\ -c_1 & c_1+c_2+c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix}$$

$$+\begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ k_3(x_2-x_3)^3 \\ -k_3(x_2-x_3)^3 \end{bmatrix} = \begin{bmatrix} 0 \\ c_2\dot{u}+k_2 u \\ 0 \end{bmatrix} \tag{1}$$