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# Complete synchronization, phase synchronization and parameters estimation in a realistic chaotic system

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ABSTRACT

The two-parameter phase space in certain nonlinear system is investigated and the chaotic region of parameters are measured to show its chaotic properties. Within the chaotic parameter region, the complete synchronization, phase synchronization and parameters estimation are discussed in detail by using adaptive synchronization scheme and Lyapunov stability theory. Two changeable gain coefficients are introduced into the controllable positive Lyapunov function and thus the parameter observers. It is found that complete synchronization or phase synchronization occurs with different controllers being used though the parameter observers are the same. Phase synchronization is observed when zero eigenvalue of Jacobi matrix, which is composed of the errors of corresponding variables in the drive and driven chaotic systems. The optimized selection of controllers can induce transition of phase synchronization and complete synchronization.



## 1. Introduction

Chaos and spatiotemporal chaos are found in the biological, physical and chemical systems, the study of chaos and spatiotemporal chaos have been paid much attention in the last decades [1–22]. For example, Misra et al. [1] discussed the role of phase synchronization in information process, Pikovsky et al. [2] investigated the phase synchronization of chaotic systems due to external forcing, Shuai et al. [3] studied the phase synchronization in certain neuron models, detailed review about different kinds of synchronization was reported in [4], Ávila et al. [5] measured the phase synchronization in the lighted-controlled oscillators, Perc et al. [6,7] gave excellent explanation about regular and chaotic calcium oscillations and control unstable orbits outside the chaotic attractor, Wu et al. [9] detected the phase synchronization and coherence resonance of calcium oscillations in coupled hepatocytes, Perc et al. [10,11] constructed the visualization of chaotic attractors and analyzed the time series of human electrocardiogram, Alatriste et al. [12] checked the phase synchronization in tilted deterministic ratchets, Nikulin et al. [13] observed the alpha and beta oscillations in the human electroencephalogram, Wei et al. [14,15] simulated the adaptive control of chaos in power system and synchronization in permanent magnet synchronous motor, Liu et al. [16] constructed a new chaotic system with three variables and give detailed presentation about its realization in circuit, Miliou et al. [17] researched the effectiveness of secure communication based on chaos synchronization in the presence of noise, Denker et al. [18] discovered the phase synchronization of the local field potential in motor cortex during movement preparation, Kim et al. [19] suggested that phase synchronization is useful to detect biological associations between genes, Bob et al. [20] confirmed the EEG phase synchronization in patients with paranoid schizophrenia,

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Erjaee and Momani [21] discussed the phase synchronization in fractional differential chaotic systems, Choi et al. [22] explored the phase synchronization between Kuramoto oscillators with finite inertia. Within the dynamical theory, nonlinear differential equations and coupled map lattices are often used to model these systems. Particularly, readers can refer to the elaborate works in this field as reported in Refs. [23,24]. Chaos and hyperchaos are observed in nonlinear differential equations when appropriate parameters are used while spatiotemporal pattern are observed in reaction–diffusion systems and coupled oscillators or networks of neurons [25–29]. It is important to measure the collective behaviors of the oscillators or neurons in the networks, particularly; the synchronization and/or the desynchronization of neuronal activities in networks of neurons could give useful clues to understand the mechanism of certain neuronal disease [30]. Within this topic, some important works should be cared, for example, Wang et al. gave elaborate presentation and investigation about time-delay and rewiring probability induced transition of synchronization in the networks of neurons with topological structure as scale-free and small-world topology type [31,32], Gosak and Perc illustrated some new ways to induce coherence resonance and stochastic resonance in the excitable networks and chaotic systems [33,34].

In recent years, many interesting chaotic systems are observed and simulated in circuits, these chaotic and/or hyperchaotic systems show different dynamical properties. These models give important information to study synchronization and secure communication [35–37]. The synchronization of chaos or hyperchaos is classified as complete synchronization [38,39], lag synchronization [40–43], generalized synchronization [44] and phase synchronization [45–50]. Transition of synchronization [51–56] can be induced in the chaotic systems and burst synchronization [57] can occur in neurons, particularly, the time-delay induced synchronization in neurons and networks reported by Wang et al. [53–56] can give good clues to understand the mechanism of information encoding and wave propagation among neurons. Furthermore, other topics about synchronization were also presented as the anti-phase synchronization [58–65] and cluster synchronization [66] and others [67]. Therefore, so many schemes have been proposed to study and detect the synchronization [47–49]. It is very critical to estimate unknown parameters in the system by using adaptive synchronization [68–70] and Lyapunov stability theory within the study and control of chaos, hyperchaos, pattern formation and transition of spatiotemporal pattern within the networks of chaotic oscillators.

In fact, some practical and realistic systems should be checked and investigated within this topic about chaos and control, for example, Krese et al. studied the dynamics of laser droplet generation in experimental and theoretical ways and it could give useful guidance for accurate welding procedures [71]. In this paper, the two-parameter region supporting chaotic state in a three-variable realistic system [16], which could be realized in circuit and give practical design for signal generator with wide broadband, wave carrier and secure keys for application in secure communication. The dynamical properties of this circuit are measured and detected by calculating the Lyapunov exponent spectrum extensively. Within the chaotic parameter region, adaptive synchronization scheme is used to study the transition of phase synchronization and complete synchronization, and the four parameters in the drive system are unknown. It is found that parameters are identified well and complete synchronization occurs with appropriate controllers and parameter observed being constructed. Phase synchronization is observed when partial unknown parameters are estimated exactly.

## 2. Problems and scheme

The three-variable chaotic system reported in [16] is described by

$$\begin{cases} dx/dt = a(z-x), \\ dy/dt = bx - xz, \\ dz/dt = xy - cy - dz, \end{cases} \tag{1}$$

where $a$, $b$, $c$, $d$ are parameters and two nonlinear terms exist in Eq. (1), chaotic state occurs when the parameters are selected with appropriate values. For example, $a = 8$, $b = 40$, $c = 10/3$, $d = 4$, Eq. (1) shows chaotic state and one positive Lyapunov exponent is approached. Extensive numerical studies are given to measure the chaotic parameter region within Eq. (1) by calculating the maximal Lyapunov exponent in the two-parameter space $a$ vs. $c$ and $b$ vs. $c$, and the results are shown in Fig. 1(a and b). Then two group of parameters are given to illustrate the chaotic attractors, the three Lyapunov exponents are 0.38833, 1.62E−4, −10.48635 at $a = 3$, $b = 40$, $c = 1.6$ and $d = 4$; and another group exponents are 0.98943, 1.52E−4, −13.97254 at $a = 5$, $c = 40$, $b = 2.5$, $d = 4$.

The results in Fig. 1(a) show that chaos can be induced in Eq. (1) as appropriate parameters $a$, $c$ are selected at fixed parameters $b = 40$, $d = 4$, and strange chaotic attractor are observed at corresponding parameter regions supporting chaos. The extensive studies are shown in Fig. 1(d) to confirm that chaos can occur by selecting appropriate parameters in the two-parameter phase space $b$ vs. $c$ at fixed parameter $a = 8$, $d = 4$.

The results in Fig. 1 show appropriate parameters could be selected to induce chaos in Eq. (1), it also indicates that chaotic time series are much different when different groups of parameters are selected to induce chaos in Eq. (1). Therefore, it is critical to detect these unknown parameters within Eq. (1) for further application and study. An improved scheme is used to study the parameter estimation, phase synchronization and complete synchronization. The corresponding driven system (response system) is given with