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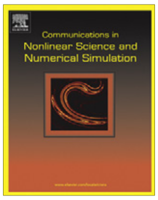

Global stability of stage-structured predator–prey models with Beddington–DeAngelis functional response

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ABSTRACT

Two stage-structured predator–prey systems with Beddington–DeAngelis functional response are proposed. The first one is deterministic. The Second one takes the random perturbation into account. For each system, sufficient conditions for global asymptotic stability are established. Some simulation figures are introduced to support the analytical findings.

1. Introduction

The dynamic relationship between prey and predator has long been and will continue to be a dominant theme in ecology due to its universal existence. One significant component of the predator–prey relationship is the predator's functional response, i.e., the rate of prey consumption by an average predator. Some biologists have argued that in many cases, especially when predators have to search for food and therefore, have to share or compete for food, the functional response in a prey–predator model should be predator-dependent. Especially, Skalski and Gilliam [1] claimed that the predator-dependent can provide better descriptions of predator feeding over a range of predator–prey abundances by comparing the statistical evidence from 19 predator–prey systems with the three predator-dependent functional responses (Hassell–Varley [2], Beddington–DeAngelis [3,4] and Crowley–Martin [5]), and in some cases, the Beddington–DeAngelis type functional response preformed even better. The classical predator–prey model with Beddington–DeAngelis type functional response is

$$\frac{dx}{dt} = x\left[b_1 - a_{11}x - \frac{a_{12}y}{1+mx+ny}\right], \quad \frac{dy}{dt} = y\left[-b_2 + \frac{a_{21}x}{1+mx+ny} - a_{22}y\right], \tag{1}$$

where $x = x(t)$ and $y = y(t)$ represent predator and prey densities at time t, respectively. b_i, a_{ij}, m, n are positive constants, i, $j = 1, 2$. For biological representation of each coefficient in the population dynamics we refer the reader to [3] and [4]. Owing to its theoretical and practical significance, (1) and its various generalized forms have received great attention lately and have been studied extensively, see e.g. [6–12] and the references therein.

System (1) assumes that each individual prey admits the same risk to be attacked by predator. This assumption is obviously unrealistic for many animals (see e.g. [13–17]). In the natural world, there are many species whose individuals have a

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life history that take them through two stages, immature and mature. Since for a number of mammals, the immature preys are concealed in the mountain cave and raised by their parents; they do not necessarily go out for seeking food, then it is reasonable to suppose that the immature prey does not have a risk to be attacked by the predator. So in this paper, based on system (1), we first propose a deterministic system with stage structure on prey. Then we take a further step by taking the environmental noise into account. The motivation is that in the real world, population dynamics is inevitably affected by the environmental noise (see e.g. [18,19]), which is an important component in an ecosystem. May [20] pointed out that due to environmental noise, the birth rates, carrying capacity, competition coefficients and other parameters involved in the system exhibit random fluctuation to a greater or lesser extent. Consequently a lot of authors introduced stochastic noise into deterministic models to reveal the effect of environmental variability on the population dynamics in mathematical ecology; see e.g. [21–32].

The paper is organized as follows. In Second two, we formulate our two predator–prey models with stage-structure on prey and Beddington–DeAngelis functional response. One is deterministic, another is stochastic. In Second three, we investigate the stability of our systems. Sufficient conditions for global asymptotic stability of each system are established. Some simulation figures are introduced to illustrate our main result in Section 4. Section 5 gives the conclusions and future directions.

2. The models

Let $x_1(t)$ stand for the population size of immature prey, $x_2(t)$ represent the population size of mature prey, $x_3(t)$ denote the population size of the predator. Firstly, we formulate the following deterministic system with stage structure on prey:

$$(M):\begin{cases}\frac{dx_1}{dt}=rx_2-d_1x_1-ax_1^2-bx_1,\\ \frac{dx_2}{dt}=bx_1-d_2x_2-a_{11}x_2^2-\frac{a_{12}x_2x_3}{1+mx_2+nx_3},\\ \frac{dx_3}{dt}=-d_3x_3+\frac{a_{21}x_2x_3}{1+mx_2+nx_3}-a_{22}x_3^2.\end{cases}$$

The model is derived under the following assumptions:

(A1) The immature prey population: the birth rate into the immature population is proportional to the existing mature prey population with proportionality $r>0$; the death rate is proportional to the existing immature prey population with proportionality $d_1>0$; overcrowding rate of the immature prey population $a>0$; the transformation rate from the immature prey to mature prey is proportional to the existing immature prey population with proportionality $b>0$.

(A2) The mature prey population: $d_2>0$ and a_{11} are the death and overcrowding rate of the mature prey population respectively; a_{12} is the capturing rate of the predator.

(A3) The predator population: the predator feed only on the mature prey. d_3 is the death rate of the predator; a_{21}/a_{12} is the rate of conversion of nutrients into the reproduction of the predator.

Let (x_1^*,x_2^*,x_3^*) ba a positive equilibrium of (M) which is the solution of the algebraic equations

$$\begin{cases}rx_2-d_1x_1-ax_1^2-bx_1=0,\\ bx_1-d_2x_2-a_{11}x_2^2-\frac{a_{12}x_2x_3}{1+mx_2+nx_3}=0,\\ -d_3+\frac{a_{12}x_2}{1+mx_2+nx_3}-a_{22}x_3=0.\end{cases}$$

Now let us take a further step by taking the environmental noise into account. Currently the many ways considered in the literatures to model the effect of the environmental fluctuations in population dynamics is to replace the parameters involved with the deterministic system by some random parameters. Recall that the parameter d_i represents the death rate. In practice we usually estimate it by an average value plus an error term. In general, by the well-known central limit theorem, the error term follows a normal distribution and is sometimes dependent on how much the current population sizes differ from the equilibrium state (see e.g. [18,19,22]). In other words, we can replace the rate d_i by an average value plus a random fluctuation term

$$d_i+\sigma_i(x_i-x_i^*)\dot{B}_i(t),$$

where σ_i^2 represents the intensity of the noise and $\dot{B}_i(t)$ is a standard white noise, namely $B_i(t)$ is a standard Brownian motion defined on a complete probability space $(\Omega,\mathcal{F},\mathcal{P})$ with a filtration $\{\mathcal{F}_t\}_{t\in R_+}$ satisfying the usual conditions (i.e., it is right continuous and increasing while $\mathcal{F}_0$ contains all $\mathcal{P}$-null sets). Then corresponding to the deterministic model system (M), the stochastic system takes the following form:

$$(SM):\begin{cases}dx_1=[rx_2-d_1x_1-ax_1^2-bx_1]dt-\sigma_1x_1(x_1-x_1^*)dB_1(t),\\ dx_2=\left[bx_1-d_2x_2-a_{11}x_2^2-\frac{a_{12}x_2x_3}{1+mx_2+nx_3}\right]dt-\sigma_2x_2(x_2-x_2^*)dB_2(t),\\ dx_3=x_3\left[-d_3+\frac{a_{21}x_2}{1+mx_2+nx_3}-a_{22}x_3\right]dt-\sigma_3x_3(x_3-x_3^*)dB_3(t),\end{cases}$$