Commun Nonlinear Sci Numer Simulat 38 (2016) 30–44

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

A bifurcation analysis of boiling water reactor on large domain of parametric spaces

Vikas Pandey, Suneet Singh*

Department of Energy Science and Engineering, Indian Institute of Technology Bombay, Mumbai 400076, India

ARTICLE INFO

Article history:
Received 12 December 2014
Revised 1 November 2015
Accepted 24 January 2016
Available online 17 February 2016

Keywords:
Dynamical system of nuclear reactors
Bifurcation of limit cycles
Limit point bifurcation of limit cycles
Period doubling bifurcation of limit cycles

ABSTRACT

The boiling water reactors (BWRs) are inherently nonlinear physical system, as any other physical system. The reactivity feedback, which is caused by both moderator density and temperature, allows several effects reflecting the nonlinear behavior of the system. Stability analyses of BWR is done with a simplified, reduced order model, which couples point reactor kinetics with thermal hydraulics of the reactor core. The linear stability analysis of the BWR for steady states shows that at a critical value of bifurcation parameter (i.e. feedback gain), Hopf bifurcation occurs. These stable and unstable domains of parametric spaces cannot be predicted by linear stability analysis because the stability of system does not include only stability of the steady states. The stability of other dynamics of the system such as limit cycles must be included in study of stability. The nonlinear stability analysis (i.e. bifurcation analysis) becomes an indispensable component of stability analysis in this scenario. Hopf bifurcation, which occur with one free parameter, is studied here and it formulates birth of limit cycles. The excitation of these limit cycles makes the system bistable in the case of subcritical bifurcation whereas stable limit cycles continues in an unstable region for supercritical bifurcation. The distinction between subcritical and supercritical Hopf is done by two parameter analysis (i.e. codimension-2 bifurcation). In this scenario, Generalized Hopf bifurcation (GH) takes place, which separates sub and supercritical Hopf bifurcation. The various types of bifurcation such as limit point bifurcation of limit cycle (LPC), period doubling bifurcation of limit cycles (PD) and Neimark–Sacker bifurcation of limit cycles (NS) have been identified with the Floquet multipliers. The LPC manifests itself as the region of bistability whereas chaotic region exist because of cascading of PD. This region of bistability and chaotic solutions are drawn on the various parametric bifurcation diagrams

1. Introduction

The nonlinear stability analysis of boiling water reactors has been of paramount interest over the last decades. The neutronics of boiling water reactors, which is responsible for the heat production in the core, is coupled with thermal hydraulics, which defines the removal of heat by the coolant from the BWRs system. The thermal hydraulic loop determines the feedback gain (reactivity feedback) and couples it with the neutronics via coefficient of void reactivity and coefficient of Doppler reactivity. This coupling and feedback mechanism induce the neutron coupled thermal hydraulic (NCTH) instability in this aforementioned system. The dynamics of these mechanisms can be mathematically represented by coupled

* Corresponding author. Tel.: +91 22 2576 7843; fax: +91 22 2576 4890.
E-mail address: suneet.singh@iitb.ac.in (S. Singh).

http://dx.doi.org/10.1016/j.cnsns.2016.01.018
1007-5704/

nonlinear ordinary differential equations. The non-linearity in the system arises due to the reactivity feedback [1,2]. The dynamical behavior predicted by linear stability analysis for these nonlinear systems is valid under certain conditions (i.e. for small perturbation). The births of the limit cycle, which are a nonlinear phenomenon, have been noticed in these systems for relatively larger perturbations in literature. These limit cycles have been noticed during stability tests and numerical calculations as well [1–4].

The three approaches of the stability analyses have been found in the literature. The frequency domain analyses are performed using the codes LAPUR, MATSTAB, NUFREQ-NP. The time domain analyses is carried out by giving one initial state of the phase space and numerically investigate the behavior of the system. RAMONA, TRAB-BWR, TRACG, RETRAN, TOSDYN-2, RELAP codes is used to for this purpose. The bifurcation analysis, which is a nonlinear stability analysis, is usually carried out for the low-dimensional model as this analysis is computationally expensive. Hence, for the qualitative study of the nuclear reactor dynamical systems, a few reduced order models (ROMs) have been developed. March–Leuba was first to develop a fifth order phenomenological reduced order model, by considering the essential features of the boiling water reactors (BWRs) [1,2]. The numerical study using this model has shown the existence of supercritical Hopf bifurcation, where a family of stable limit cycles excites from the Hopf point, leads system to a chaotic solution via cascading of period doubling bifurcations of limit cycles. However, this analysis was limited to a few parametric spaces (i.e. only for feedback gain) and only for detection of aperiodic solutions. The analytical bifurcation analysis using the March–Leuba model has been done by Munoz-Cobo and Verdu [5] using the Poincare normal form analytical technique. The supercritical Hopf points have been studied in this work and results are verified with those obtained by March–Leuba [1,2]. Apart from these works, unstable limit cycles have been also observed in the work of Tsuji and bifurcation theory have been used for the detection of subcritical Hopf bifurcations [6]. This work proposed another model of BWR, which includes neutron kinetics and heat transfer along with thermal hydraulics in coolant channel. The computer code BIFOR2 has been utilized for the analysis [7]. The bifurcation of limit cycles was a lesser concern in this field of the BWR dynamical system. However, these type of bifurcations have been discussed in the work of Rizwan-uddin [8]. This extended nonlinear stability analysis investigated sub and supercritical Hopf bifurcation. The turning point bifurcation of limit cycles was briefly discussed in work, which is also called as limit point bifurcation of limit cycles (LPCs). This study is based on the semi-analytical bifurcation analysis, which has been carried out with the bifurcation code BIFFDD [20]. Recently, a modified March–Leuba model has been considered by Bindra and Rizwan-uddin, which includes the impact of the change in void reactivity with the change in temperature. However, only linear stability analysis has been carried out in this work [9]. A simplified mathematical model for boiling water reactor has been proposed by Wahi and Kumawat to study the stability of the system by the method of multiple scales for specific values of advanced heavy water reactor (AHWR) [10]. The stability analysis of this model has been extended by Pandey and Singh to identify the coexistence of bistability in the system (one stable fixed point and one large amplitude stable limit cycle) [11]. The recent developments in nonlinear stability analysis of the NPPs have shown the existence of unstable limit cycles in the linear stable regions [12–14]. The limit point bifurcation of limit cycles has also been observed. Hence, for the complete physical understanding of the nuclear reactor, it is necessary to identify the regions where the unstable limit cycles co-exist with the stable fixed points. The occurrence of LPC shows that the stable limit cycles of large amplitude co-exist with an unstable limit cycle. The present work adopts March–Leuba model [1] to the identification of subcritical and supercritical Hopf bifurcation and generalized Hopf bifurcation (GH). Adding to this, bifurcation of limit cycles and origination of LPC curves from GH points has been studied. It is pointed out that the existing literature on this model has shown the existence of period doubling bifurcation of limit cycles and limit point bifurcation of limit cycles for a particular point. However, in the present work, a series of LPC and PD curve have been drawn in different parametric regimes. The regions bounded by these LPC curves are the region of bistability where one stable fixed point and one stable limit cycles co-exist. The regions covered by PD curves are the regions in which the solution manifolds can be aperiodic as mentioned by March–Leuba. A numerical continuation bifurcation package MatCont has been used here for the whole analysis [15].

The aforementioned work in this paper has been presented in five sections. In Section 1 generalized introduction about the problem and state of the art is included. Section 2 contains the mathematical background of the bifurcation theory, which is given for better understanding to the general readers. The criterion for the Hopf bifurcation has been reproduced. The significance of mathematical coefficients has been presented in this section as well. A brief overview of March–Leuba [1] model for BWR is reproduced in Section 3. The numerical continuation is applied in Section 4 to determine the codimension-1 and codimension-2 bifurcation of fixed points and codimension-1 bifurcation of limit cycles for the different parametric spaces. At last, in Section 5, conclusions have been made for the study.

2. Mathematical background

The goal of the present work is to investigate complicated dynamics (limit cycles, bistability) for aforementioned boiling water reactors. A brief review of bifurcation routes and the related mathematical background is given in this section for the convenience of a general reader, for a detail study one can refer [16].

2.1. Hopf bifurcation: a codimension-1 bifurcation of fixed point

The codimension refers the number of free parameters for a bifurcation to occur in the system. Hopf bifurcation, a codimension-1 bifurcation, is the only bifurcation of its category, which have been observed in this work. Hence, it is obvious