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Anomalous convection diffusion and wave coupling transport of cells on comb frame with fractional Cattaneo–Christov flux

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ABSTRACT

An improved Cattaneo–Christov flux model is proposed which can be used to capture the effects of the time and spatial relaxations, the time and spatial inhomogeneous diffusion and the spatial transition probability of cell transport in a highly non-homogeneous medium. Solutions are obtained by numerical discretization method where the time and spatial fractional derivative are discretized by the L1-approximation and shifted Grünwald definition, respectively. The solvability, stability and convergence of the numerical method for the special case of the Cattaneo–Christov equation are proved. Results indicate that the fractional convection diffusion-wave equation is an evolution equation which displays the coexisting characteristics of parabolicity and hyperbolicity. In other words, for α in (0, 1), the cells transport occupies the characteristics of coupling convection diffusion and wave spreading. Moreover, the effects of pertinent time parameter, time and spatial fractional derivative parameters, relaxation parameter, weight coefficient and the convection velocity on the anomalous transport of cells are shown graphically and analyzed in detail.

1. Introduction

The fractional order partial differential equations have attracted much attention in science research and engineering and also played an important role in the fields of biology and medicine [1]. One of the mathematical models used to imitate cells transport is called comb model which can be described by the time fractional Fokker–Planck equation [2]. The geometry of comb model [3], as shown in Fig. 1, includes backbone and fingers, which are used to mimic cells transport. The dissemination of cells includes two independent processes: cell fission and cell transport, namely the proliferation and migration dichotomy [4] is considered. As shown in Refs. [5,6], a special behavior of comb model is that the displacement in the x direction only happens along the x axis. Thus, it is supposed that the diffusion coefficient in the x direction is $D_{xx} = D\delta(y)$ while the diffusion coefficient along the y direction is $D_{yy} = D_0$. Moreover, the convection velocity along the x direction is considered as $u\delta(y)$.

Recently a considerable attention has been devoted to the study of the diffusion on a comb frame. Baskin and Iomin [7] studied the specific properties of cells transport, i.e., super-diffusion, on a comb structure, it was shown that an inhomogeneous convection flow was a mechanism for the realization of the Lévy-like process. A frontier case of super-diffusion, where the transport exponent approaches infinity, was studied. The toy model of fractional transport of cancer cells due

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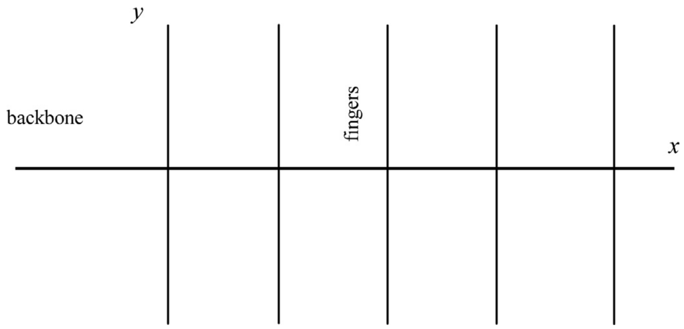

Fig. 1. Schematic drawing of the comb model.

to self-entrapping was proposed in [8], the super-diffusion of cancer on a comb structure was studied in [9]. Santamaria et al. [10] analyzed the anomalous diffusion in Purkinje cell dendrites caused by spines. The diffusion of a volume marker, fluorescein dextran, within spiny dendrites was remarkably slow in comparison to its diffusion in smooth dendrites, the computer simulations indicated that this retardation was due to a transient trapping of molecules within dendritic spines and they can be described by the diffusion in comb frame. Arkhincheev et al. [11] studied the random walk and capture of active species responsible for various processing modifications of porous low dielectric constant materials, the results can be applied for the description of various technological processes including participation of active species. Zahran et al. [12] also studied the fractional Fokker–Planck equation using a comb-like structure. For the force-free case, the distribution function associated with space dependence diffusion coefficient along the backbone of the structure was obtained in a closed form of H-function. For the critical literatures about the comb model, see Refs. [13–15].

In view of the above mentioned works, the 1-D constitutive equation used in describing the cells transport on a comb frame is based on the time fractional Fick's first law of diffusion [16,17], namely the time-nonlocal dependence between the flux j and the gradient function $\frac{\partial P}{\partial x}$, given by:

$$j = -D_{RL}^{1-\alpha}\left(D\frac{\partial P}{\partial x}\right), \quad 0 < \alpha \le 1 \tag{1}$$

and the corresponding continuity equation is written as:

$$\frac{\partial P}{\partial t} + \mathrm{div}\, j = 0 \tag{2}$$

where j and D refer to the diffusion flux and diffusion coefficient, respectively. $P(x,t)$ is a new distribution function which is defined by $P(x,t) = \int_{-\infty}^{\infty} P_1(x,y,t)dy$ where $P_1 = P_1(x,y,t)$ is the distribution function at the special location (x,y) and time t. The classical comb model corresponds with the fractional parameter $\alpha = 1/2$. The symbol $D_{RL}^{1-\alpha}$ is the Riemann–Liouville fractional derivative [18] of order $1-\alpha$, given by:

$$D_{RL}^{1-\alpha}P(x,t) = \frac{\partial}{\partial t}\left[\frac{1}{\Gamma(\alpha)}\int_0^t \frac{1}{(t-\tau)^{1-\alpha}}P(x,\tau)d\tau\right], \quad 0 < \alpha \le 1. \tag{3}$$

The Fick's first law is a most successful model in describing the diffusion problems in various pertinent situations. However, a serious limitation is that it contradicts with the principle of causality due to the fact that any initial disturbance is felt instantly throughout the entire medium [19].

Cattaneo [20] proposed a modified constitutive model which overcame the shortcoming of Fick's model and took the finite velocity of propagation into account [21]. On the basis of Eq. (1), the time fractional Cattaneo constitutive model to describe the cells transport on a comb frame can be given as follows:

$$j + \xi\frac{\partial j}{\partial t} = -D_{RL}^{1-\alpha}\left(D\frac{\partial P}{\partial x}\right), \tag{4}$$

where the propagation velocity [21] v is defined by $(D/\xi)^{1/2}$, ξ is a nonnegative constant and refers particularly to the relaxation time of diffusion, $\xi \to 0$ and $\alpha = 1$ corresponds to the classical Fick's first law of diffusion with an infinite velocity. Qi and Guo [22] studied the effects of Cattaneo flux on the transient fractional heat conduction with specific initial boundary value conditions and obtained the exact solution in terms of the H-functions. Atanacković et al. [23] considered the space-time Cattaneo heat conduction law that contains the Caputo symmetrized fractional spatial derivative and fractional time derivative, the explicit form of the solution was obtained and some limiting cases were shown graphically. For more literatures related to the Cattaneo model, see Refs. [24,25].

The Cattaneo flux model merely involves the partial time derivative. In reality, it is well known that, for a more complete formulation, both the partial time derivative and spatial derivative should be taken into account. Christov improved the