Commun Nonlinear Sci Numer Simulat 38 (2016) 130–139

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Two dimensional gap solitons in self-defocusing media with PT-symmetric superlattice

Hongcheng Wang[a,b,*], D.N. Christodoulides[b]

[a] College of Electronic Engineering, Dongguan University of Technology, Dongguan 523808, China
[b] CREOL, The College of Optics and Photonics, University of Central Florida, Orlando, FL 32816-2700, USA

ARTICLE INFO

Article history:
Received 14 October 2015
Revised 13 February 2016
Accepted 15 February 2016
Available online 26 February 2016

Keywords:
Optical solitons
Parity-time symmetry
Optical lattice
Linear stability

ABSTRACT

A theory is presented to study the existence and stability properties of fundamental solitons, multi-peaked gap solitons and vortex solitons in self-defocusing media with PT-symmetric superlattice. In the first gap of the superlattice potential, fundamental, dipole, and in-phase quadruple solitons can exist stably within a wide parameter region, while in-phase two-peaked solitons, out-of-phase quadruple solitons and vortex solitons are always unstable. Compared with simple-lattice solitons in similar potentials, the gap solitons in superlattice have a wider stable range than those in simple-lattice.

1. Introduction

Recently light propagation in nonlinear optical media with parity-time (PT) symmetric potential has attracted much research interest since optical PT-synthetic materials could lead to some new phenomena which is impossible in standard optical systems [1]. This interest was motivated by various areas of physics, including quantum field theory and mathematical physics [2]. In quantum mechanism, spectra can be entirely real in a wide class of non-Hermitian Hamiltonians if the system presents PT symmetry [3]. This result obtained in quantum mechanics can be directly analogy to the field of optics because they share the same mathematical formalism, named nonlinear Schrödinger equation. To present PT-symmetry, a two-dimensional (2D) optical potential $V(x,y)$ must satisfy the condition $V^*(x, y) = V(-x, -y)$, where (x, y) are the normalized transverse coordinates, and $*$ represents complex conjugation. Such PT-symmetric structures can be achieved in optics by an appropriate modulation of the complex refractive index, that is, $n^*(x, y) = n(-x, -y)$ [4–6]. In other words, the real part of the index profile should be symmetric in space while the imaginary component (gain–loss) is anti-symmetric. This kind of structures can be used to enforce stable single-mode operation in micro-ring laser resonators [7] and to observe some interesting phenomena, such as double refraction and optical solitons [8–19]. In 2008, beam dynamics in PT-symmetric complex-valued periodic optical structures was first studied for both one- and two-dimensional geometries, and fundamental PT-symmetric solitons were proved to be stable over a wide range of potential parameters [5]. Up to now, PT-symmetric solitons and their stability properties have been widely studied in various optical nonlinearities with various kinds of optical lattices, including Scarff II potential [5], Gaussian potential [8], simple periodical lattice with or without defects [9–12], and Bessel potential etc. [13,14]. It has been proved that stable multi-hump solitons and vortex solitons can exist in nonlinear Kerr media with 2D PT-symmetric optical lattices [15–17]. Recently, PT-symmetric and anti-symmetric fundamental solitons

* Corresponding author. Tel.: +86 15920689122.
E-mail address: hc_wang@126.com (H. Wang).

http://dx.doi.org/10.1016/j.cnsns.2016.02.013

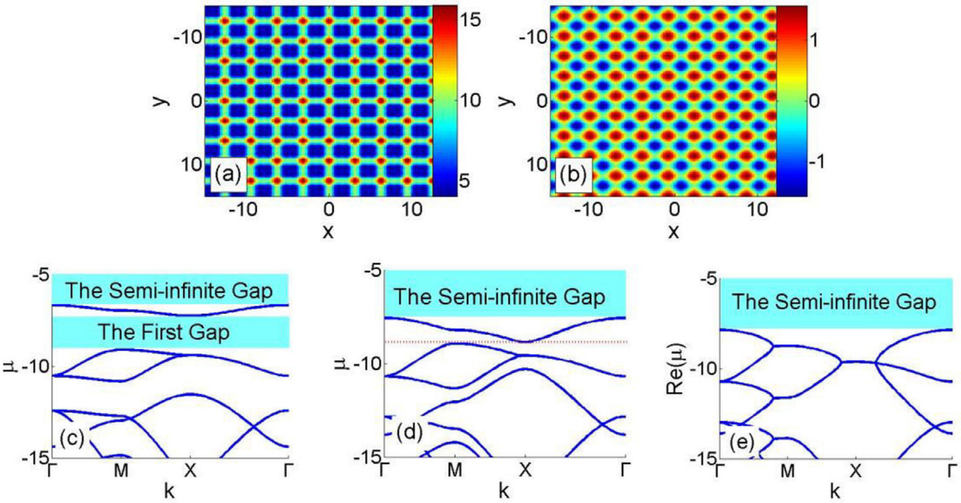

Fig. 1. (Color online) The profiles of (a) real part and (b) imaginary part of a typical two-dimensional PT-symmetric superlattice. (c–e) Diffraction relations of PT-symmetric superlattices for three W_0 values of (c) 0.8, (d) 1.98, and (e) 2.2 at $\varepsilon_1 = 0.7$, and $V_0 = 8$. The dotted line in (d) shows the merge of the first band and the second band.

(FSs) and on-site-centered solitary vortices were also found in a 2D network built of PT-symmetric dimers with on-site cubic nonlinearity, the gain and loss elements of the dimers being linked by parallel square-shaped lattices [18]. Usually, optical solitons have different optical field profiles and stability properties in different optical lattice. Optical superlattice has been proved to be able to exhibit collective properties not shared by either constituent, and therefore optical solitons in nonlinear media with superlattice can present different properties from those in simple lattice, which include the concavity in the central of soliton profile [19] and the Y-shape propagation [20]. Very recently, gap solitons have been proved to exist in two-dimensional mixed linear–nonlinear complex optical lattices and the nonlinear lattice modulation has great effect on the existence and stability [21]. Though optical solitons in nonlinear Kerr media with optical superlattice have been reported, it still remains an open question if the stable gap solitons can exist in self-defocusing media with two-dimensional superlattices. In this paper, we study the existence and stability properties of fundamental, multi-pole, and vortex solitons in defocusing Kerr media with PT-symmetric superlattice. In this model, the real part of the superlattice potential is achieved by adding spatial harmonic potential with double period to a simple lattice potential with basic period. It is found that superlattice can help stabilize the fundamental solitons and multipeaked solitons formed in the first gap.

2. Theoretical model

Beam propagation in self-defocusing Kerr nonlinearity with two-dimensional PT-symmetric optical lattice can be described by the following normalized nonlinear Schrödinger equation:

$$i\frac{\partial U}{\partial z} + \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - V(x, y)U - |U(x, y)|^2 U = 0, \tag{1}$$

where U is the slowly varying amplitude of the beam, z is the normalized longitudinal coordinate, x and y are the normalized coordinates along the transverse directions. In the case of the 2D PT-symmetric optical superlattices, the complex potential $V(x,y)$ is described by:

$$V(x, y) = V_0\left[\varepsilon\left(\cos^2 x + \cos^2 y\right) + (1-\varepsilon)\left(\cos^2 2x + \cos^2 2y\right)\right] + iW_0(\sin 2x + \sin 2y),$$

where V_0 and W_0 are the depth of the real and imaginary parts of PT-symmetric lattices, respectively. ε is a real parameter controlling the relative strength of superlattice, and $0 < \varepsilon < 1$. The real and imaginary parts of a typical superlattice potential when $\varepsilon = 0.7$, $V_0 = 8$ and $W_0 = 0.8$ are shown in Fig. 1(a) and Fig. 1(b), respectively.

For better understanding of the bifurcation of the gap solitons, we first study the band structure of this optical superlattice. The continuous spectrum of this nonlinear Schrödinger equation consists of Bloch modes of the form $U(x, y, z) = F(x, y)\exp[i(k_x x + k_y y)]\exp(i\mu z)$, where $F(x,y)$ is periodic function in (x,y), and μ is the propagation constant, k_x and k_y are the wave numbers in the first Brillouin zone. The diffraction relation $\mu = \mu(k_x, k_y)$ can be obtained from the direct substitution of into the linear version of Eq. (1) and numerical solving by the plane wave expansion method. The band structure at $V_0 = 8$, $W_0 = 0.8$, and $\varepsilon = 0.7$ is plotted in Fig. 1(c), where the Γ-point represents the center of the first Brillouin zone, whereas the M-point and X-point are the band edges. Similarly as in the case of simple PT-symmetric lattice, there exists a semi-infinite gap $\mu \geq -6.6714$ (semi-infinite region above the first band), together with the first gap $-9.1059 \leq \mu \leq -7.2548$ (the finite region between the first band and the second band). The second gap vanishes because the overlap of the second