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Different synchronization characteristics of distinct types of traveling waves in a model of active medium with periodic boundary conditions

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ABSTRACT

The model of a one-dimensional active medium, which cells are the FitzHugh–Nagumo oscillators, is studied for periodical boundary conditions. The medium possesses three different regimes in dependence on the parameter values. The regimes correspond to the self-sustained oscillations, excitable dynamics or bistability of the medium cells. Periodic boundary conditions provide the existence of traveling wave modes in all mentioned cases without any deterministic or stochastic excitation. The spatial waveforms and the character of oscillations in time can be similar in the different cases, but the properties of wave modes depend considerably on the medium regime. So, the dispersion characteristics and the synchronization phenomena are essentially different for bistable and excitable media on the one hand, and for the self-sustained oscillatory medium on the other hand. The local and distributed periodic influence on the medium are studied. The phenomenon of the traveling wave frequency locking is observed for all three regimes of the active medium. The comparison of synchronization effects in self-oscillatory, excitable and bistable regimes of the active medium is carried out.

1. Introduction

The study of the active media behavior remains one of the actual trends in nonlinear dynamics today. Spatially distributed excitable systems and media that are widely spread in the biophysical problems are of special interest [1–5]. In many cases a medium can be represented as a distributed system consisting of a very large number of locally coupled small size elements. There are three types of active media, i.e., self-sustained oscillatory media, excitable media and bistable media. The elements of these media are respectively self-sustained oscillators, excitable systems and bistable systems with two equilibrium points [6,7]. The auto-wave processes are observed in active media, i.e., the damped traveling waves propagate in space. They are the phase waves in self-sustained oscillatory media, the waves of excitation and switching in the excitable and bistable media, respectively. But only a self-sustained active medium demonstrates absolute instability and stationary oscillations are observed in any point of unbounded space for any initial and boundary conditions. In order to support the oscillation process in cases of excitable and bistable media without external force and noise, the additional conditions are necessary to provide the return of excitation emerged in some element back to the same element. The examples include spiral waves of excitation in the space [8–11] or traveling waves arising in a ring of excitable oscillators or in a one-dimensional

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excitable medium with periodic boundary conditions [3,12–16]. Traveling waves of switching in a ring of bistable oscillators are also known, but they have been explored insignificantly. When the most popular model of a bistable system, namely, the Duffing oscillator, is used as an element of a medium, then the wave modes do not emerge in the case of diffusive coupling. Unidirectional (convective) coupling may result in the appearance of traveling waves in the ring [17,18]. But this type of coupling is not so typical for the natural systems as diffusion interaction. Unidirectional coupling provides the additional energy input to the system, that can produce complex oscillations [19].

Thus, traveling waves can emerge in an active medium with periodic boundary conditions for different character of medium elements. This fact is known but a number of problems still remain unstudied. At first, the question arises: is the repeating oscillatory process in elements of excitable and bistable media a special kind of self-sustained oscillations? It is known that one of the fundamental features of self-sustained oscillations is the frequency synchronization, i.e., the locking of characteristic frequencies under external influence or as a result of interaction of several systems [20].

The phenomenon of stochastic synchronization has been found for excitable and bistable systems, whose oscillations are induced by noise for some conditions (coherent resonance, stochastic resonance). A large number of scientific papers was devoted to this phenomenon, for example [21–23]. However, the synchronization effect for deterministic traveling waves in the excitable and bistable media remains significantly less studied. There are a number of articles devoted to the control of the oscillation period (interspike interval) in neuron models which represent a ring of excitable systems [24–26]. These results show that the local external force can change the frequency of excitation pulses. Thus, the phenomenon of frequency locking takes place. However, these works were not aimed specifically to study the properties of synchronization. In particular, the existence of synchronization area while the parameters of external excitation are varied has not been considered. The features inherent to the synchronization of traveling waves in excitable and bistable distributed systems and media and their differences from self-sustained oscillatory systems hitherto remain unexplored.

Results of numerical simulation of a one-dimensional active medium consisting of FitzHugh–Nagumo oscillators [27,28] are presented in this work. This medium can be self-sustained, excitable and bistable one depending on the values of parameters. Periodic boundary conditions provide the existence of traveling wave modes in self-sustained, excitable and bistable regimes. The external excitation of the medium by a periodic force is studied. The cases of local and spatially-distributed excitations are considered. The goal of this work has been to study the traveling waves and their synchronization in the self-sustained, excitable and bistable media with periodic boundary conditions. We try to answer the following questions:

1. Are the traveling waves in deterministic excitable and bistable media the special kinds of self-sustained oscillations as well as waves in self-sustained oscillatory medium?
2. What are the differences between synchronization effects in the three mentioned cases of the medium?

2. Model of the medium

The active medium consisting of identical FitzHugh–Nagumo oscillators is being studied. The FitzHugh–Nagumo oscillator is a simplified neuron model [27,28] and is widely used in nonlinear dynamics and biophysics as an example of the simplest excitable system. However, along with an excitable regime the FitzHugh–Nagumo oscillator can have a bistable regime with two stable equilibrium points or self-sustained oscillations depending on the values of parameters. For all these reasons the FitzHugh–Nagumo oscillator is the most suitable element for active medium simulation. The medium regime can be controlled by changing the values of parameters. The FitzHugh–Nagumo oscillator is described by the following equations:

$$\begin{cases} \varepsilon \dfrac{dx}{dt} = x - y - \alpha x^3, \\ \dfrac{dy}{dt} = \gamma x - y + \beta, \end{cases} \tag{1}$$

where $x = x(t), y = y(t)$ are dimensionless real dynamic variables; t is dimensionless time; α, β, γ, ε are system parameters.

The bifurcation diagram of the system (1) on the plane of control parameters γ, β is shown in Fig. 1. The diagram illustrates the bistable area and the area of self-sustained oscillations. The bistable area boundary corresponds to a saddle-node bifurcation of equilibrium points and does not depend on the parameter ε. The self-sustained oscillations boundary is the Andronov–Hopf bifurcation line whose position is changed with the change of parameter ε. The excitable regime is observed in the area of the only stable equilibrium point near the Andronov–Hopf bifurcation line under the condition that the parameter ε is small. In this case, the oscillator (1) with noise excitation demonstrates a well-known coherence resonance effect [29,30]. The diagram in Fig. 1 does not show all the details of the bifurcation portrait of system (1) in the vicinity of the intersection of the lines of saddle-node bifurcations and Andronov–Hopf bifurcations (close to $\beta = 0$ and $\gamma = 1$). We omit the description of the details, as they are quite complex and are not directly related to the problem of the article.

Consider N oscillators (1) united in a one-dimensional ring by means of local connections. Such a distributed system can be described by the following equations: