Commun Nonlinear Sci Numer Simulat 38 (2016) 277–287

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Nonlinear ion acoustic waves scattered by vortexes

Yuji Ohno*, Zensho Yoshida

Graduate School of Frontier Sciences, The University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, Chiba 277-8561, Japan

ARTICLE INFO

Article history:
Received 1 December 2015
Revised 23 February 2016
Accepted 25 February 2016
Available online 5 March 2016

Keywords:
Nonlinear ion acoustic wave
Vortex
Reductive perturbation method
Painlevé analysis

ABSTRACT

The Kadomtsev–Petviashvili (KP) hierarchy is the archetype of infinite-dimensional integrable systems, which describes nonlinear ion acoustic waves in two-dimensional space. This remarkably ordered system resides on a singular submanifold (leaf) embedded in a larger phase space of more general ion acoustic waves (low-frequency electrostatic perturbations). The KP hierarchy is characterized not only by small amplitudes but also by irrotational (zero-vorticity) velocity fields. In fact, the KP equation is derived by eliminating vorticity at every order of the reductive perturbation. Here, we modify the scaling of the velocity field so as to introduce a vortex term. The newly derived system of equations consists of a generalized three-dimensional KP equation and a two-dimensional vortex equation. The former describes ‘scattering’ of vortex-free waves by ambient vortexes that are determined by the latter. We say that the vortexes are ‘ambient’ because they do not receive reciprocal reactions from the waves (i.e., the vortex equation is independent of the wave fields). This model describes a minimal departure from the integrable KP system. By the Painlevé test, we delineate how the vorticity term violates integrability, bringing about an essential three-dimensionality to the solutions. By numerical simulation, we show how the solitons are scattered by vortexes and become chaotic.

1. Introduction

The ion acoustic waves (IAWs) serve as a rich source of nonlinear phenomena. The combination of the nonlinearity (by fluid convection) and the dispersion (by the nonlocal electric interactions) enables IAWs to produce various structures ranging from order (such as solitons) to chaos (turbulences). At the simplest one-dimensional geometry, small-amplitude IAWs become solitons; Washimi and Taniuti [1] derived the Korteweg–de Vries (KdV) equation by the reductive perturbation method. The Kadomtsev–Petviashvili (KP) equation [2] is a two-dimensional generalization of the KdV equation, which was picked up by the ‘Kyoto School’ as the archetype of infinite-dimensional integrable systems [3,4]. Kako and Rowlands [5] derived three types of two-dimensional generalizations of Washimi and Taniuti’s result, including the KP equation. Diverse directions of generalizations have been also studied; for example, variations of the KdV equation including finite ion temperature [6,7], multi-ions [8], and dust plasma [9]; as well as variations of the KP equation including multi-ions [10], dust plasma [11], and multi-temperature [12,13]. The modifications to include third-order nonlinear terms were proposed by considering trapped electrons [14,15]. Effects of higher order terms in the reductive perturbation method have been also widely studied (see, e.g., Refs. [9,16,17] and references therein). The higher order perturbations are called clouds, and the solitons are called dressed solitons.

* Corresponding author. Tel.: +81 471363990.
E-mail address: ohno@ppl.k.u-tokyo.ac.jp (Y. Ohno).

http://dx.doi.org/10.1016/j.cnsns.2016.02.018
1007-5704/

In this paper, we explore a new direction of generalization – we introduce a *vorticity* to the system, and delineate a fundamental change of dynamics brought about by the vortex. The KP hierarchy is characterized not only by small amplitudes but also by irrotational (zero-vorticity) velocity fields (see Section 2.2). We may view the ordered system of solitons as a singular submanifold (leaf) embedded in a larger phase space of finite-vorticity perturbations [18]. The departure from the zero-vorticity leaf will produce complexity and, finally, generate turbulence. The aim of this study is to probe into the 'neighborhood' of the KP hierarchy and elucidate how chaos starts to develop.

We organize this paper as follows. In Section 2, we show that the KP equation is derived by eliminating vorticity at every order of the reductive perturbation. We also show that the reductive perturbation succeeds only if the entropy is homogeneous; hence the baroclinic effect, a creation mechanism of vorticity, must be absent (Appendix A). In Section 3, we introduce a new ordering of velocity field in order to formulate a finite-vorticity system. The new system is composed of a generalized three-dimensional KP equation and a two-dimensional vorticity equation. The former describes 'scattering' of vortex-free waves by ambient vortexes that are determined by the latter. We say that the vortexes are 'ambient' because they do not receive reciprocal reactions from the waves. In Section 4, we invoke the Painlevé test to study whether the new system is integrable or not. The result is negative. By this analysis, we elucidate that the scattering by the vorticity introduces an essential three-dimensionality in the wave fields, by which the integrability condition (in the sense of the Painlevé test) is broken. In Section 5, we perform numerical simulations to visualize how chaos occurs. Section 6 concludes our investigations.

2. Reductive perturbation method for Kadomtsev–Petviashvili equation and vorticity

2.1. Kadomtsev–Petviashvili equation

We start by remembering the derivation of the KP equation by the *reductive perturbation method* [1,2,5]. The basic equations for nonlinear IAWs are expressed as

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\boldsymbol{u}) = 0, \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = \nabla\phi, \tag{2}$$

$$-\Delta\phi = n - \mathrm{e}^{\phi}, \tag{3}$$

where n is the ion number density, $\boldsymbol{u} = (u, v, w)^{\top}$ is the ion velocity, ϕ is the electrostatic potential, and Δ is the Laplacian. We consider cold ions and adiabatic electrons with a constant temperature T_{e}. The variables are normalized as followings: the density n by a representative density n_0, the velocity $\boldsymbol{u}$ by the ion sound speed $c_s = \sqrt{T_{\mathrm{e}}/m_{\mathrm{i}}}$ (where m_{i} is the ion mass), the electrostatic potential ϕ by the characteristic potential T_{e}/e, the coordinate variable $\boldsymbol{x}$ by the Debye length $\sqrt{\varepsilon_0 T_{\mathrm{e}}/n_0 e^2}$, and the time variable t by the ion plasma frequency $\sqrt{n_0 e^2/\varepsilon_0 m_{\mathrm{i}}}$.

We consider IAWs propagating in two-dimensional space (x, y). The extension to three-dimensional space (x, y, z) will be discussed later. We assume that waves propagate primarily in the direction of x, and introduce a set of stretched variables

$$\tilde{x} = \epsilon(x - t), \qquad \tilde{y} = \epsilon^2 y, \qquad \tilde{t} = \epsilon^3 t, \tag{4}$$

with a small parameter ϵ. The dependent variables n, ϕ, u, and v are expanded as

$$\begin{cases} n = 1 + \epsilon^2 n_1 + \epsilon^4 n_2 + \cdots, \\ \phi = 0 + \epsilon^2\phi_1 + \epsilon^4\phi_2 + \cdots, \\ u = 0 + \epsilon^2 u_1 + \epsilon^4 u_2 + \cdots, \\ v = 0 + \epsilon^3 v_1 + \epsilon^5 v_2 + \cdots. \end{cases} \tag{5}$$

From the terms of orders ϵ^2 and ϵ^3, we obtain $n_1 = \phi_1$ and $\partial n_1/\partial\tilde{x} = \partial u_1/\partial\tilde{x} = \partial\phi_1/\partial\tilde{x}$. Assuming the boundary conditions $n_1, \phi_1, u_1 \to 0$ $(x \to \pm\infty)$, we put

$$n_1 = u_1 = \phi_1. \tag{6}$$

From the terms of order ϵ^4, we obtain

$$\frac{\partial v_1}{\partial\tilde{x}} = \frac{\partial\phi_1}{\partial\tilde{y}} \tag{7}$$

and $n_2 = \phi_2 + \phi_1^2/2 - \partial^2\phi_1/\partial\tilde{x}^2$. From the terms of order ϵ^5, we obtain the two-dimensional KP equation:

$$\frac{\partial}{\partial\tilde{x}}\left(\frac{\partial\phi_1}{\partial\tilde{t}} + \phi_1\frac{\partial\phi_1}{\partial\tilde{x}} + \frac{1}{2}\frac{\partial^3\phi_1}{\partial\tilde{x}^3}\right) + \frac{1}{2}\frac{\partial^2\phi_1}{\partial\tilde{y}^2} = 0. \tag{8}$$