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Short communication

Solitary and periodic waves in two-fluid magnetohydrodynamics

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ABSTRACT

A system of equations of two-fluid magnetohydrodynamics is studied. An ordinary differential equation describing traveling waves in an ideal cold quasi-neutral plasma is obtained in the case of quasi-stationary electromagnetic field. The Painlevé analysis of this equation is carried out and the general solution of the equation is constructed in terms of the Weierstrass elliptic function. Solitary and periodic wave solutions for the components of magnetic field are found and analyzed.

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1. Introduction

The electrons and ions in the two-fluid magnetohydrodynamics (MHD) are considered as interpenetrating conducting liquids. It is also supposed that the main hydrodynamic parameters (a mass density, speed and pressure) depend on spatial coordinates and time. A current density and magnetic field are used as electromagnetic parameters. Taking into account the Maxwell equations, heat transfer equations and state equations for the electrons and ions, one can obtain a closed system of equations (see, e.g. [1–3]).

This mathematical model is widely used in astrophysics for description of solar corona [4–7], plasma influenced by the gravitational field of black holes [8] and in others applications [9–19]. However, these works was mainly devoted either to the linear or long wave approximations or numerical investigation of mathematical models of the two-fluid MHD. Recently, an attempt to study this model analytically was made in [20] and solitary traveling wave solutions for the magnetic field were found. In this work we generalize these results and find the general traveling wave solution of this mathematical model of the two-fluid MHD.

The rest of this work is organized as follows. In the next section we present a closed system of equation for the description of an ideal cold plasma in the two fluid approximation. Then we introduce traveling wave variables and transform this system of equations into a system of two ordinary differential equations. Section 3 is devoted to constructing and analyzing the general solution of this system of equations. In the last section we briefly summarize our results.

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2. Equations of two-fluid electromagnetic hydrodynamics (EMHD) of plasma

Let us consider the motion of an ideal cold plasma, so the heat transfer and state equations are not taken into account. In addition, we assume that the plasma is quasi-neutral and the electromagnetic field is quasi-stationarity. In this case to describe motion of the plasma the following system of equations can be used [1,20,21]:

$$\begin{aligned}
&\frac{\partial n_e}{\partial t}+\mathrm{div}(n_e\mathbf{V_e})=0,\\
&\frac{\partial n_i}{\partial t}+\mathrm{div}(n_i\mathbf{V_i})=0,\\
&m_e n_e\left(\frac{\partial \mathbf{V_e}}{\partial t}+(\mathbf{V_e},\nabla)\mathbf{V_e}\right)=-en_e\left(\mathbf{E}+\frac{1}{c}[\mathbf{V_e},\mathbf{B}]\right),\\
&m_i n_i\left(\frac{\partial \mathbf{V_i}}{\partial t}+(\mathbf{V_i},\nabla)\mathbf{V_i}\right)=en_i\left(\mathbf{E}+\frac{1}{c}[\mathbf{V_i},\mathbf{B}]\right),\\
&\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}+\mathrm{rot}\mathbf{E}=0,\quad \mathrm{div}\mathbf{B}=0,\\
&\mathrm{rot}\mathbf{B}=\frac{4\pi}{c}\mathbf{j},\quad \mathbf{j}=e(n_i\mathbf{V_i}-n_e\mathbf{V_e}),\\
&n_e=n_i=n.
\end{aligned}\tag{2.1}$$

Here, n_e, n_i are concentrations of electrons and ions and $\mathbf{V_e}$, $\mathbf{V_i}$ are their velocities, $\mathbf{E}$ is an electric field, $\mathbf{B}$ is an magnetic field, $\mathbf{j}$ is a current density, t is time, ∇ is the Nabla operator. The first two equations in (2.1) are continuity equations of electrons and ions, the third and fourth equations are equations of motion for electrons and ions, the next four equations are the quasi-stationary Maxwell equations and the last equation expresses the assumption of the quasi-neutrality of plasma.

System of Eq. (2.1) is a complex system of nonlinear partial differential equations. However, this system of equations can be considerably simplified if we use the following variables [22,23]:

$$\rho=m_e n_e+m_i n_i,\tag{2.2}$$

$$\mathbf{U}=\frac{m_e\mathbf{V_e}+m_i\mathbf{V_i}}{m_e+m_i},\tag{2.3}$$

where $\mathbf{U}$ is a mass hydrodynamic velocity of the plasma and ρ is a mass density of the plasma. We also consider one-dimensional motion of the ideal cold plasma assuming that $\partial/\partial y=\partial/\partial z=0$.

As a result, system (2.1) takes the form:

$$\begin{aligned}
&\frac{\partial\rho}{\partial t}+\frac{\partial\rho U_x}{\partial x}=0,\\
&\frac{\partial\rho U_x}{\partial t}+\frac{\partial}{\partial x}\left(\rho U_x^2+\frac{1}{4\pi}\frac{B_y^2+B_z^2}{2}\right)=0,\\
&\frac{\partial\rho U_y}{\partial t}+\frac{\partial}{\partial x}\left(\rho U_x U_y-\frac{1}{4\pi}B_xB_y\right)=0,\\
&\frac{\partial\rho U_z}{\partial t}+\frac{\partial}{\partial x}\left(\rho U_x U_z-\frac{1}{4\pi}B_xB_z\right)=0,\\
&E_y-\frac{c^2m_em_i}{4\pi e^2\rho}\frac{\partial^2E_y}{\partial x^2}=\frac{1}{c}(B_zU_x-B_xU_z)-\frac{cm_em_i}{4\pi e^2\rho}\frac{\partial}{\partial x}\left(U_x\frac{\partial B_z}{\partial x}\right)+\frac{m_i-m_e}{4\pi e\rho}B_x\frac{\partial B_y}{\partial x},\\
&E_z-\frac{c^2m_em_i}{4\pi e^2\rho}\frac{\partial^2E_z}{\partial x^2}=\frac{1}{c}(B_xU_y-B_yU_x)+\frac{cm_em_i}{4\pi e^2\rho}\frac{\partial}{\partial x}\left(U_x\frac{\partial B_y}{\partial x}\right)+\frac{m_i-m_e}{4\pi e\rho}B_x\frac{\partial B_z}{\partial x},\\
&B_x=const,\\
&E_x=\frac{1}{c}(B_yU_z-B_zU_y)-\frac{m_i-m_e}{4\pi e\rho}\frac{\partial}{\partial x}\left(\frac{B_y^2+B_z^2}{2}\right),\\
&\frac{1}{c}\frac{\partial B_y}{\partial t}-\frac{\partial E_z}{\partial x}=0,\\
&\frac{1}{c}\frac{\partial B_z}{\partial t}+\frac{\partial E_y}{\partial x}=0.
\end{aligned}\tag{2.4}$$

Thus, we get closed system of Eq. (2.4) with respect to the variables ρ, U_x, U_y, U_z, E_y, E_z, B_y, B_z.

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