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# Attitude stabilization of rigid spacecraft with disturbance generated by time varying uncertain exosystems

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ABSTRACT

This paper is concerned with the problem of attitude stabilization of rigid spacecraft in the presence of parameter uncertainty and external disturbance generated by some time varying uncertain exosystems. A time varying internal model that is free of the uncertain parameter is proposed to compensate the disturbance. An adaptive attitude controller is presented by incorporating time varying dynamic coordinate transformation, semi-tensor product and adaptive control method. Especially, the parameter uncertainties caused by the inertia matrix and uncertain exosystems are tackled by combining the semi-tensor product and adaptive control method. Finally, a simulation example is presented to illustrate the effectiveness of the proposed method.



## 1. Introduction

In recent years, the attitude control problem of rigid spacecraft has gained extensive interest from the control systems community. In order to enhance the performance of closed-loop systems, various nonlinear control methods have been proposed under various assumptions and scenarios [1–10].

In [1], an attitude control design framework was proposed for the analysis of attitude control of a rigid body using the non-singular unit quaternion representation. In [11], an adaptive control scheme for the attitude control of a rigid spacecraft was derived from passivity-based adaptive control schemes, and global convergence of the tracking error to zero was shown. In [12], a passivity-based control without angular velocity feedback was further proposed to achieve the asymptotical convergence of the attitude tracking. In [13], the author presented a continuous globally stable tracking control for rigid spacecraft based on a variable structure control design. The adaptive control method was used in [14] to deal with model uncertainties. In [15], by combining the adaptive control and optimal control, an inverse optimal adaptive control was presented to achieve attitude tracking and disturbance attenuation for a class of disturbances with bounded energy. In [16], the continuous finite-time control and terminal sliding mode control technique have been employed to stabilize spacecraft attitude. In [17], by combining the sliding mode control and adaptive control, attitude stabilization of rigid spacecraft with finite time convergence was considered. In [18], the attitude control problem of a rigid spacecraft was converted into a global stabilization problem, and a dynamic compensator based on internal model method was designed to deal with the external disturbances, which are sum of finitely many sinusoidal functions with the frequencies are known, and the designed controller can achieve disturbance rejection; while the same problem was considered in [19] for the case where the frequencies of the disturbance are unknown. Note that, the disturbances

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assumed in [18,19] can be generated by some linear autonomous exosystems. Recently, in [20], a robust adaptive control approach based on the time varying internal model and semi-tensor product(STP) theory [21] was presented for the attitude control and disturbance rejection problem of rigid spacecraft with the disturbances are generated by some time-varying exosystems. It is worth noting that, the advantage of the internal model based control is that it can achieve asymptotical disturbance rejection, not just disturbance attenuation, and the advantage of using the STP to deal with the parameter uncertainty is that the STP can overcome the non-commutative shortage of the conventional matrix product.

However, the problem of attitude control and disturbance rejection of rigid spacecraft has not been studied when the disturbances are generated by some uncertain time varying exosystems. In this paper, motivated by [20] and [22], we will further consider a more interesting and realistic problem where a rigid spacecraft involves inertia matrix parameter uncertainty and external disturbances which are generated by some uncertain time varying exosystems. It is worth noting that, the problem studied in [20] can be viewed as a special case of this paper since the exosystems in [20] do not involve any uncertainty. On the other hand, the techniques proposed in [22] can not be used directly to solve our problem since [22] studied the output regulation problem of a class of single input single output systems, while the system studied in this paper is a multi-input multi-output system. Therefore, to overcome these difficulties, we need to develop a different technique. The main contributions of this paper are as follows: (1) the new matrix product, i.e., semi-tensor product, is applied to deal with parameter uncertainties in the inertia matrix and the complicated uncertainties caused by the unknown inertia matrix and the uncertain exosystems; (2) the proposed internal model, dynamic coordinate transformation, semi-tensor product, and backstepping are incorporated to construct an adaptive attitude control law for the rigid spacecraft.

The rest of this paper is organized as follows. Section 2 introduces some standard assumptions, the problem formulation, and some preliminaries of the STP of matrices. In Section 3, a time varying internal model is proposed to deal with the external disturbances. In Section 4, an adaptive control law is presented based on the proposed internal model, dynamic coordinate transformation, the STP and backstepping control. A numerical simulation example is shown in Section 5 to verify the effectiveness of the proposed method. Finally, some concluding remarks follow in Section 6.

## 2. Spacecraft attitude model and problem formulation

For a rigid spacecraft, the attitude kinematics and dynamics can be described as follows [15,23]:

$$\begin{aligned}
\dot{q}_v &= \frac{1}{2}(q_4 I_3 + q_v^{\times})\omega, \\
\dot{q}_4 &= -\frac{1}{2}q_v^T\omega, \\
J\dot{\omega} &= -\omega^{\times}J\omega + u + d,
\end{aligned} \tag{1}$$

where $\mathbf{q} = (q_v, q_4) \in \mathbb{R}^3 \times \mathbb{R}$ is the unit quaternion representing the attitude orientation of the spacecraft with the constraint $q_v^T q_v + q_4^2 = 1$, $q_v = [q_1, q_2, q_3]^T$ and $q_4$ denote the vector part and scalar part of the unit quaternion, respectively. $\omega(t) \in \mathbb{R}^3$ is the angular velocity of the body fixed reference frame $\mathcal{F}$ with respect to an inertial reference frame $\mathcal{I}$, $I_3$ is the $3 \times 3$ identity matrix, and $v^{\times}$ denotes the cross product operator for a vector $v = [v_1, v_2, v_3]^T$ defined by

$$v^{\times} = \begin{pmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{pmatrix}.$$

$J$ is a constant, positive definite, symmetric inertia matrix, $u = [u_1, u_2, u_3]^T \in \mathbb{R}^3$ is the control input torque, $d = [d_1, d_2, d_3]^T \in \mathbb{R}^3$ represents the external disturbance, which is generated by the following time varying uncertain exosystems,

$$\begin{aligned}
\dot{v}_i &= a_i(t, \sigma, v_i), \\
d_i(t, \sigma) &= H_i v_i(t, \sigma), \ \ i = 1, 2, 3,
\end{aligned} \tag{2}$$

where $v_i \in \mathbb{R}^{n_{v_i}}$, $\sigma \in \mathbb{S} \subset \mathbb{R}^{n_\sigma}$ with $\mathbb{S}$ is a known compact subset of $\mathbb{R}^{n_\sigma}$, $H_i$ is a $1 \times n_{v_i}$ matrix.

To have our problem well posed, we assume the following:

**Assumption 1.** The inertia matrix $J$ appeared in Eq. (1) is unknown and can be represented by

$$J = J_0 + \Delta J, \tag{3}$$

where $J_0$ is the nominal part of $J$ and $\Delta J$ is the uncertain part of $J$.

**Assumption 2.** For each $i = 1, 2, 3$, there exist positive integers $r_i$ and sufficiently smooth functions $a_{i,1}(t, \sigma)$, $a_{i,2}(t, \sigma), \ldots, a_{i,r_i}(t, \sigma)$, such that

$$\frac{d^{r_i} d_i(t, \sigma)}{dt^{r_i}} = \frac{d^{(r_i-1)}(a_{i,r_i}(t, \sigma) d_i(t, \sigma))}{dt^{(r_i-1)}} + \cdots + \frac{d(a_{i,2}(t, \sigma) d_i(t, \sigma))}{dt} + a_{i,1}(t, \sigma) d_i(t, \sigma). \tag{4}$$

**Remark 3.** It should be pointed out that Assumption 2 is motivated by [22,24]. And, the Eq. (4) will reduce to the equation (14) in [18] when the sufficiently smooth functions $a_{i,1}(t, \sigma), a_{i,2}(t, \sigma), \ldots, a_{i,r_i}(t, \sigma)$ are all constants. On the other hand, the