Commun Nonlinear Sci Numer Simulat 35 (2016) 166–176

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Stochastic responses of a viscoelastic-impact system under additive and multiplicative random excitations

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ARTICLE INFO

Article history:
Received 14 April 2015
Revised 6 November 2015
Accepted 12 November 2015
Available online 21 November 2015

Keywords:
Viscoelastic-impact system
Non-smooth transformation
Stochastic averaging method
Stochastic bifurcation

ABSTRACT

This paper deals with the stochastic responses of a viscoelastic-impact system under additive and multiplicative random excitations. The viscoelastic force is replaced by a combination of stiffness and damping terms. The non-smooth transformation of the state variables is utilized to transform the original system to a new system without the impact term. The stochastic averaging method is applied to yield the stationary probability density functions. The validity of the analytical method is verified by comparing the analytical results with the numerical results. It is invaluable to note that the restitution coefficient, the viscoelastic parameters and the damping coefficients can induce the occurrence of stochastic P-bifurcation. Furthermore, the joint stationary probability density functions with three peaks are explored.

1. Introduction

A large number of viscoelastic materials have received tremendous attention these years, such as polymers, composite materials and metals. These materials that are widely used in mechanical engineering, aerospace engineering and civil engineering can both store and dissipate system energy. However, it is a challenge to develop constitutive models to describe viscoelastic characteristics. Over the last few decades, many models [1–3] have been developed and explored, such as fractional derivative model, Kelvin–Voigt model and so on. Commonly, a simple linear viscoelastic model based on the generalized Maxwell model [4,5] has been widely studied.

The nonlinear dynamics of the viscoelastic system that are subject to the deterministic case [6–9] have been discussed widely in the previous literatures. Since stochastic perturbations are ubiquitous in nature and society [10], it is necessary to investigate the behavior of the viscoelastic system under stochastic excitations. In recent years, Soize and Poloskov [11] investigated the transient response of linear viscoelastic systems with model uncertainties and stochastic excitation by a time-domain formulation. Huang and Xie [12] applied the first-order and second-order stochastic averaging method to study the stochastic stability of a SDOF linear viscoelastic system under the excitation of wideband noise. Floris [13] analyzed the stochastic stability of a hinged-hinged viscoelastic column. By approximating a viscoelastic force as a damping force and a stiffness force, Zhu and Cai [14] discussed the stochastic responses of a viscoelastic system with strongly nonlinear stiffness force under broadband random excitations. Through using the largest Lyapunov exponent method, the stability of a viscoelastic system under wideband noise excitations has been suggested by Ling [15].

A great many non-smooth factors exist very naturally in engineering applications [16], such as impacts, dry frictions and collisions. Due to the existence of the non-smooth factors, the theories of smooth systems cannot be used to the non-smooth

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http://dx.doi.org/10.1016/j.cnsns.2015.11.008
1007-5704/

systems directly. As an important type of the non-smooth system, the vibro-impact system has been extensively explored. Huang [17] studied the stationary responses of a multi-degree-of-freedom vibro-impact system under white noise excitations according to the Hertz contact law. By introducing the mean Poincaré map, Feng [18,19] obtained the mean response of impact systems. Then, Dimentberg [20–22] and Iourtchenko [23,24] obtained impact energy losses for linear vibration systems with impact and the response probability density functions (PDFs) of stochastic linear systems with impact numerically. Refs. [25–29] extended the stochastic averaging method to exhibit the stochastic response of the vibro-impact systems based on the non-smooth transformation.

More and more facts indicate that the viscoelastic models can make a more accurate description and give a deeper insight into the inherent nature of realistic physical systems. In addition, a large number of practical problems in mechanics and engineering can be modeled as impact systems due to the existence of impacts and collisions. Clearly, most previous researches focused on the responses of the vibro-impact systems or the viscoelastic systems. To the best of our knowledge, a few studies on the responses of the viscoelastic-impact systems [30] are well known and none of scholars considered the stochastic responses of a viscoelastic-impact system under additive and multiplicative random excitations. Therefore, much more efforts are needed to investigate a viscoelastic-impact system under random excitations.

Based on the above discussion, this paper explores the stochastic responses of a viscoelastic-impact system under additive and multiplicative random excitations by using the stochastic averaging approach. The paper is organized as follows. Section 2 considers replacement of the viscoelastic force and non-smooth transformation. Then, the stochastic averaging method is applied to obtain the approximate stationary PDFs in Section 3. Section 4 discusses the validity of the method is verified by numerical simulation results and stochastic bifurcations. The conclusions are summarized in Section 5.

2. Replacement of the viscoelastic force and non-smooth transformation

In this section, by introducing an approximate procedure, the viscoelastic force is first replaced by a combination of stiffness and damping terms. Then based on the non-smooth transformation, the state variables are utilized to transform the original system to a new system without the impact term.

2.1. Replacement of the viscoelastic force

Consider the following viscoelastic-impact system under additive and multiplicative random excitations,

$$\ddot{x} + Z + \varepsilon(c_3x^4 - c_2x^2 - c_1)\dot{x} + \omega_0^2 x = \varepsilon^{1/2}\xi_1(t) + \varepsilon^{1/2}x\xi_2(t),\ x > 0, \tag{1a}$$

$$\dot{x}_+ = -r\dot{x}_-,\ x = 0, \tag{1b}$$

where $c_1, c_2, c_3, \varepsilon$ and ω_0 are constants. $\xi_1(t)$ and $\xi_2(t)$ are Gaussian white noises with zero mean and independent correlations, which satisfy

$$\langle \xi_i(t)\rangle = 0, R_{ij}(\tau) = \langle \xi_i(t)\xi_j(t+\tau)\rangle = \begin{cases} 2D_{ij}\delta(\tau), & \mathrm{i} = \mathrm{j}, \\ 0, & \mathrm{i} \neq \mathrm{j}, \end{cases} \quad (\mathrm{i}, \mathrm{j} = 1, 2). \tag{2}$$

where $\langle \bullet \rangle$ denotes the averaging operation about t.

Here D_{11} and D_{22} are the intensity of the noises $\xi_1(t)$ and $\xi_2(t)$, respectively. $\delta(\tau)$ is the Dirac delta function. In Eq. (1b), $\dot{x}_+$ and $\dot{x}_-$ refer to the value of response velocity after and before the impact, respectively. Restitution factor r $(0 < r \leq 1)$ indicates energy loss during impact. Obviously, the impact loss of the system becomes very small when r approaches to 1. It is particularly to note that the impact loss of the system is zero with $r = 1$. The viscoelastic force Z in Eq. (1a) is as follows:

$$Z[x(t)] = \int_0^t h(t-\tau)x(\tau)d\tau, \tag{3}$$

where $h(t)$ is the relaxation function for the generalized Maxwell model, which is given by

$$h(t) = \sum_i \beta_i \exp(-t/\lambda_i), \lambda_i \geq 0, \tag{4}$$

Eqs. (3) and (4) manifest that total viscoelastic force Z may be composed of a large number of viscoelastic components, in which λ_i denotes the relaxation time, and β_i is known as the general elastic modulus. Substituting Eq. (4) into Eq. (3), the viscoelastic force is replaced by the following expression:

$$Z = \int_0^t \sum_i \beta_i \exp[-(t-\tau)/\lambda_i]x(\tau)d\tau. \tag{5}$$

According to Liu and Zhu [31], when ε is proportional to small parameters, $X(t-s)$ can be approximated as follows:

$$X(t-s) \approx X\cos\bar{\omega}s - \frac{\dot{X}}{\bar{\omega}}\sin\bar{\omega}s, \tag{6}$$