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Short communication

Modeling the self-similarity in complex networks based on Coulomb's law

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ABSTRACT

Recently, self-similarity of complex networks have attracted much attention. Fractal dimension of complex network is an open issue. Hub repulsion plays an important role in fractal topologies. This paper models the repulsion among the nodes in the complex networks in calculation of the fractal dimension of the networks. Coulomb's law is adopted to represent the repulse between two nodes of the network quantitatively. A new method to calculate the fractal dimension of complex networks is proposed. The Sierpinski triangle network and some real complex networks are investigated. The results are illustrated to show that the new model of self-similarity of complex networks is reasonable and efficient.

1. Introduction

Network science is widely used in many academic research [1–4]. Recently, complex networks have attracted much attention in diverse areas [5–8]. Representation by complex networks of complex systems has been proved to be generally successful to describe their various features [9–11]. It has been shown that the small-world [12] property and the scale-free [13] property are the two fundamental properties of complex networks.

Fractal theory is applied to model complex systems [14,15]. The strict self-similarity property or the statistical self-similarity property is the typical characteristic of the fractal objects. Fractal dimension is regarded as an intrinsic characteristic used to describe the complexity of a fractal object [14]. Fractal and self-similarity have been found widely exist in nature [16–18]. Song *et al.* found that a variety of real complex networks consist of self-repeating patterns on all length scales in Ref. [19]. Recently, fractal and self-similarity of complex networks have attracted much attention [20–24]. Many researchers have analyzed the fractal property of complex networks and proposed different algorithms to calculate the fractal dimension of complex networks [10,25–30]. Wei *et al.* studied the self-similarity of weighted complex networks [31]. Due to the essential uncertain in the real world [32–35], fuzzy set theory [36,37] is also applied to determine the fractal dimension of complex systems [38].

In real application, the accurate value of the fractal dimension defined by Hausdorff [14] cannot be obtained. The box-covering algorithm is a commonly used tool to calculate the fractal dimension, namely box dimension. The box-covering algorithm is to use the minimal number of boxes to cover the fractal objects. The number of boxes, denoted by $N(\varepsilon)$, and the size of the box,

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denoted by ε, yield the relationship: $N(\varepsilon) \sim \varepsilon^{D_B}$, D_B is called the box dimension, it can be obtained by regressing $\log(N(\varepsilon))$ vs. $\log(\varepsilon)$. The box-covering algorithm is widely used to calculate the fractal dimensions of complex networks [39,40].

The hubs are very special and usually play an important role in many academic research [31,41,42]. According to Song's research [43], during the self-similar dynamical evolution of complex networks, the hubs grow by preferentially linking with less-connected nodes to generate a more robust fractal topology. In other words, the nodes with high degree do not connect directly and dislike to being in the same box. It seems that a higher degree of hub repulsion plays a more important role in fractal topologies than non-fractals. As a result, it is necessary and reasonable to model the repulsion among the nodes in the complex networks in calculation of the fractal dimension of the networks, which is our motivation of this paper. In Coulomb's law [44], if the two charges have the same sign, the electrostatic force between them is repulsive. Inspired by the modeling of repulsion in Coulomb's law, we adopt this law to represent the repulse between two nodes quantitatively. In our proposed model, the connected nodes in the complex networks are regarded as "electric charges" and there exists "electrostatic interaction force" between them over the edge. The value of the repulsive force of each edge relies on the degrees of the two nodes linked by this edge. The connected nodes with higher degree have greater force over their link, thus they will be less likely to be covered by the same box in the box-covering process.

The rest of this paper is organized as follows. In Section 2, some basic concepts are introduced. The new model of complex network inspired by Coulomb's law and the proposed method to obtain the fractal dimension are proposed in Section 3. Some applications are illustrated in Section 4. Finally, the conclusions are drawn in the last section.

2. Preliminaries

In this section, some basic concepts of complex network, the typical box-covering algorithm for fractal dimension of complex network and Coulomb's law are introduced.

For a complex network $G = (N, E)$ and $N = (1, 2, \ldots, n)$, $E = (1, 2, \ldots, m)$, where n is the total number of the nodes, m is the total number of the edges, and the cell $e_{ij}(i, j = 1, 2, \cdots, n)$ of the edges equals 1 if node i is connected to node j, and 0 otherwise.

Degree is a basic indicator [45], the degree of node i, denoted by k_i, equals the number of nodes which are connected to node i directly. Degree describes the number of neighbors a node has. Shortest path between node i and node j, denoted by S_{ij}, is a set of edges which can connect node i and node j and satisfy the number of such edges are minimized [46]. Shortest path length between node i and node j, denoted by l_{ij}, is the minimized number of such edges which connect node i and node j. The diameter of a network, denoted by d, is the longest shortest path length among any two nodes.

The original definition of box-covering is initially proposed by Hausdorff [15]. It is initially applied in the complex networks by Song, *et al.* [19,39]. For a given box size l_B, a box is a set of nodes where all shortest path length l_{ij} between any two nodes i and j in the box are smaller than l_B. The minimum number of boxes used to cover the entire network, denote by N_B, yield the following relationship:

$$N_B \sim {l_B}^{-D_B} \tag{1}$$

D_B is the box dimension of complex network G, it can be obtained by regressing $\log(N_B)$ vs. $\log(l_B)$.

In Song's box-covering algorithm, the length of every edge equals to 1 and the shortest path length between any two nodes is the minimum number of edges which connect them. The maximum shortest path length between any two nodes in the box cannot be greater than the box size. The fractal dimension is calculated via a greedy coloring algorithm. There are several main steps of Song's method showing as follows. An example of Song's method [39] is illustrated in Fig. 1.

Transforming For given network G_1 and box size l_B. A new network G_2 is obtained, in which node i is connected to node j when $l_{ij} \geq l_B$.

Greedy coloring Mark as much as possible nodes in network G_2 with the same color, but the color of every node must be different from the colors of its nearest neighbors. A coloring network G_3 can be obtained.

Box counting The nodes in the same color are in the same box, so the number of boxes equals the number of colors, the value of N_B is obtained.

Regressing Change the box size l_B, D_B can be obtained by regressing $\log(N_B)$ vs. $\log(\mathrm{l_B})$.

This paper adopts Coulomb's law to represent the repulse between two nodes, so Coulomb's law is briefly reviewed. Suppose e_1 and e_2 are two point charges, with electric charges q_1 and q_2 respectively and their distance r, and then the electrostatic interaction force between them, denoted by F could be calculated by [44]

$$F = k_e \frac{|q_1 q_2|}{r^2} \tag{2}$$

If the two charges have the same sign, the electrostatic force between them is repulsive; if they have different sign, the force between them is attractive. Coulomb's law states that the magnitude of the Electrostatics force of interaction between two point charges is directly proportional to the scalar multiplication of the magnitudes of charges and inversely proportional to the square of the distances between them.