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# Modeling of chaotic motion of gyrostats in resistant environment on the base of dynamical systems with strange attractors

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ABSTRACT

A chaotic motion of gyrostats in resistant environment is considered with the help of well known dynamical systems with strange attractors: Lorenz, Rössler, Newton–Leipnik and Sprott systems. Links between mathematical models of gyrostats and dynamical systems with strange attractors are established. Power spectrum of fast Fourier transformation, gyrostat longitudinal axis vector hodograph and Lyapunov exponents are find. These numerical techniques show chaotic behavior of motion corresponding to strange attractor in angular velocities phase space. Cases for perturbed gyrostat motion with variable periodical inertia moments and with periodical internal rotor relative angular moment are considered; for some cases Poincaré sections areobtained.



## 1. Introduction

Problem of rigid bodies motion and its practical engineering applications such as gyroscopes, gyrostats and dual-spin-spacecraft are very important for modern science. Despite classical analytical research results and exact solutions this problem is still far from complete due to the existence of chaos phenomena [1–13]. Among the basic directions of modern research within the framework of the indicated problem it is possible to highlight the following points: deriving exact and approximated analytical and asymptotic solutions, investigation into stability of motion, the analysis of motion under an influence of external regular and stochastic disturbance, research into dynamic chaos and study of non-autonomous systems with variable parameters.

Recently, chaotic dynamic has becomes one of the major part of nonlinear science. Applications of dynamical systems with chaotic behavior and strange attractors are seen in many areas of science, including space-rocket systems [7–12]. Lorenz and Rössler systems [1,2] represent classical dynamical systems with strange attractors. Leipnik and Newton [3] found two strange attractors in rigid body motion. Since Leipnik and Newton's work, the chaotic dynamics of rigid body motion investigates in many works. Sprott [4,5] examined 19 systems of three-dimensional autonomous ordinary differential equations with strange attractors; also critical points, Lyapunov exponents and fractional dimensions of systems were found.

Work [7] contains the analysis of chaotic behavior of a spacecraft with periodic time-dependent moments of inertia during its free motion. The equations of variable mass coaxial bodies system were developed in papers [10] where also the atti-

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tude motion of coaxial bodies system and double rotation spacecraft with time-dependent moments of inertia were analyzed on the base of special method of phase trajectory curvature analysis. The results [7–12] can be used for the analysis of attitude motion of a gyrostat-satellites and dual-spin spacecraft including motion with an active solid-propellant rocket engine.

In this paper more attention is focused on chaotic attractors in phase space of angular velocity of gyrostat and on perturbed gyrostat motion in resistant environment with energy dissipation/excitation.

Conditions of correspondence of mathematical models of gyrostats in resistant environment and dynamical systems with strange attractors (Lorenz, Rössler, Newton–Leipnik and Sprott) are defined. To confirm the system chaotic behavior numerical computer simulations are used. These simulations are performed by means of numerical integration of the equations of motion with the help of several numerical tools: time history of phase coordinates, gyrostat longitudinal axis vector hodograph, Poincaré map, fast Fourier transform power spectrum. This characterizes the dynamical behavior of the gyrostat in resistant environment as regular or chaotic.

## 2. Mathematical model

Let us consider a gyrostat attitude motion about fixed point in resistant environment with energy dissipation/excitation (Fig. 1). Assume resistant environment effect corresponding to action of external forces moments that are constant $(\mathbf{M}^e_{const})$, linear $(\mathbf{M}^e_{lin})$ and nonlinear $\left(\mathbf{M}^e_{quad}\right)$ in main body angular velocity projections onto body frame axes $x_1x_2x_3$ $\left(\boldsymbol{\omega} = [p, q, r]^T\right)$.

The motion equations follow from angular moment's law:

$$\dot{\mathbf{K}} + \boldsymbol{\omega} \times (\mathbf{K} + \mathbf{R}) = \mathbf{M}^e_{const} + \mathbf{M}^e_{lin} + \mathbf{M}^e_{quad}, \tag{1}$$

where

$$\begin{aligned}
&\mathbf{K} = \mathbf{I} \cdot \boldsymbol{\omega}; \quad \mathbf{R} = [R_1, R_2, R_3]^T; \quad \mathbf{M}^e_{const} = [d_1, d_2, d_3]^T; \quad \mathbf{M}^e_{lin} = \mathbf{A} \cdot \boldsymbol{\omega};\\
&\mathbf{M}^e_{quad} = \mathbf{B} \cdot \left[p^2, q^2, r^2\right]^T; \quad \mathbf{A} = [a_{ij}]; \quad \mathbf{B} = [b_{ij}];\\
&a_{ij} = const; \quad b_{ij} = const; \quad R_i = const; \quad d_i = const; \quad i,j = 1, \ldots, 3.
\end{aligned} \tag{2}$$

**K** – angular moment of gyrostat main body with "frozen" internal rotor; **I** – inertia tensor of main body with "frozen" internal rotor; **R** – constant angular moment of relative rotor motion (in body frame); **A**, **B** – constant matrixes.

Matrix structure of external forces moments (2) can describe an action of viscous drag, hydro (aero) dynamic lift, nonuniform lift and friction in fluid flow ($\mathbf{V}_0$) of main body with roughened surface and propeller elements.

Assume coincidence of gyrostat center of mass, rotor center of mass and fixed point. Also let us consider case of spherical inertia tensor of rotor and gyrostat general inertia tensor $\mathbf{I} = diag(A,B,C)$. In this case scalar form of Eq. (1) can be write as follows:

$$\begin{cases}
A\dot{p} = (B - C)rq + a_{11}p + (a_{12} - R_3)q + (a_{13} + R_2)r + b_{11}p^2 + b_{12}q^2 + b_{13}r^2 + d_1,\\
B\dot{q} = (C - A)pr + a_{22}q + (a_{23} - R_1)r + (a_{21} + R_3)p + b_{21}p^2 + b_{22}q^2 + b_{23}r^2 + d_2,\\
C\dot{r} = (A - B)qp + a_{33}r + (a_{31} - R_2)p + (a_{32} + R_1)q + b_{31}p^2 + b_{32}q^2 + b_{33}r^2 + d_3.
\end{cases} \tag{3}$$

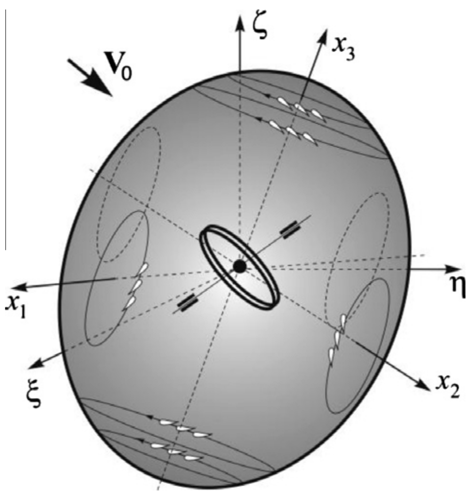


**Fig. 1.** Inertial ($\xi\eta\zeta$) and gyrostat main body ($x_1x_2x_3$) frames.