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Multi-stable chaotic attractors in generalized synchronization

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In this work, for given driving and response systems, the phenomenon of multi-stable chaotic attractors existing in generalized synchronization is studied. Consider the driving system descried by a Rössler system, and the response system being a multi-scroll chaotic system, some numerical simulations are proposed. The results show that by choosing suitable coupled parameters, there are multi-stable chaotic attractors in the response system, and each of them synchronizes with the driving system. Moreover, the basins of attraction on the parameter plane and initial condition plane are analyzed.

1. Introduction

Since Pecora and Carroll realized synchronization of chaotic systems [1], chaos synchronization has attracted much attention [2–8]. At present, several types of chaos synchronization have been revealed, such as complete synchronization (CS), phase synchronization (PS), lag synchronization (LS), anticipated synchronization (AS) and generalized synchronization (GS). In brief, CS means that systems have the same trajectory, while GS denotes that there exists a map between two systems. Therefore, GS extends the notion of CS and is a more important topic, which includes many synchronization phenomena observed in laboratory experiments [9–13]. In 1995, Rulkov et al. first described the GS phenomenon and presented the idea of mutual false nearest neighbors to detect the GS [14]. In 1996, Abarbanel et al. suggested the auxiliary system method to study GS in driving-response systems [15], and the theory about this method was given in Ref. [16]. According to the auxiliary system approach, we can determine whether there has been GS. Firstly, generate a system similar to the response system, which is called the auxiliary system. It is worth noting that initial conditions of them should be different. Then, choose the same and suitable coupled parameters in these two systems, when they realize CS, the driving system and response system will realize GS.

It is known that the chaotic systems depend on parameters and initial conditions sensitively, which maybe induce multi-stability arising in the process of synchronization. In recent years, some references have investigated relation between the multi-stability and synchronization, such as Refs. [17–22]. In this paper, taking two multi-scroll chaotic systems for illustration [23], we further research the existence of multi-stable chaotic attractors in the GS. Lü et al. introduced the multi-scroll chaotic systems in Refs. [23–26], which are complex systems and maybe have applications in such fields as secure and digital communications, and so on. However, we just provide some numerical simulations here, for that it is very difficult to provide theoretical analysis at present. As can be seen from the simulation results, when the coupled parameters satisfy certain conditions, the response system does have multi-stable attractors, and all of them synchronize with the driving system.

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Moreover, the number of the attractors equals that of the scrolls in the response system. Additionally, we analyze the basins of attraction of attractors.

2. Multi-stable chaotic attractors in the GS

Take the Rössler system with time scaling factor [22] as following for the driving system:

$$\begin{cases} \dot{x}_1 = -\alpha(x_2 + x_3), \\ \dot{x}_2 = \alpha(x_1 + a_1 x_2), \\ \dot{x}_3 = \alpha[b_1 + x_3(x_1 - c_1)], \end{cases} \tag{1}$$

where the parameters are set as $a_1 = b_1 = 0.2$, $c_1 = 5.7$, and α is the time scaling factor, which is introduced to control the characteristic time scale, or to adjust the natural frequency of rotation of the Rössler system.

In the following subsections, two modified Lorenz systems in Ref. [23] are chosen as the response system, which have three or four scrolls respectively.

2.1. Numerical simulation of three-scroll chaotic system

In this subsection, consider the response system described as:

$$\begin{cases} \dot{y}_1 = \frac{1}{3}[-(a_2+1)y_1 + (a_2 - c_2 + y_3)y_2] + \frac{1}{3\sqrt{y_1^2+y_2^2}}\left[(1-a_2)(y_1^2 - y_2^2) + 2(a_2 + c_2 - y_3)y_1 y_2\right] + k_1(x_1 - y_1), \\ \dot{y}_2 = \frac{1}{3}[(c_2 - a_2 - y_3)y_1 - (a_2+1)y_2] + \frac{1}{3\sqrt{y_1^2+y_2^2}}\left[2(a_2-1)y_1 y_2 + (a_2 + c_2 - y_3)(y_1^2 - y_2^2)\right] + k_2(x_2 - y_2), \\ \dot{y}_3 = \frac{1}{2}(3y_1^2 y_2 - y_2^3) - b_2 y_3 + k_3(x_3 - y_3). \end{cases} \tag{2}$$

When $a_2 = 10$, $b_2 = 8/3$, $c_2 = 28$, and the coupled parameters $k_1 = k_2 = k_3 = 0$, the graphical representation of the system (2) is shown as Fig. 1, we can see that the system (2) has a three-scroll chaotic attractor.

To verify GS between the driving system (1) and response system (2), according to the aforementioned auxiliary system method, we firstly generate the auxiliary system of the response system (2), which is described as the system (3):

$$\begin{cases} \dot{z}_1 = \frac{1}{3}[-(a_2+1)z_1 + (a_2 - c_2 + z_3)z_2] + \frac{1}{3\sqrt{z_1^2+z_2^2}}[(1-a_2)(z_1^2 - z_2^2) + 2(a_2 + c_2 - z_3)z_1 z_2] + k_1(x_1 - z_1), \\ \dot{z}_2 = \frac{1}{3}[(c_2 - a_2 - z_3)z_1 - (a_2+1)z_2] + \frac{1}{3\sqrt{z_1^2+z_2^2}}[2(a_2-1)z_1 z_2 + (a_2 + c_2 - z_3)(z_1^2 - z_2^2)] + k_2(x_2 - z_2), \\ \dot{z}_3 = \frac{1}{2}(3z_1^2 z_2 - z_2^3) - b_2 z_3 + k_3(x_3 - z_3). \end{cases} \tag{3}$$

By means of simulation, it is shown that for given α, when $k_3 = 0$, there exist suitable parameters k_1 and k_2, which can cause the response system (2) and auxiliary system (3) to be CS. However, there are no multi-stable chaotic attractors. For instance, when the initial conditions of the systems (1)–(3) are taken as (0.1,0.2,0.3), (0.5,1,1.5) and (2,2.5,3), respectively, and $\alpha = 5$, $k_1 = 5$, the results are shown from Figs. 2, 3. Only one chaotic attractor of the response system (2) can be seen in Fig. 2. Furthermore, Fig. 3 demonstrates that synchronization errors between the systems (2) and (3) asymptotically reach zero, which means that the response system (2) and auxiliary system (3) realize CS.

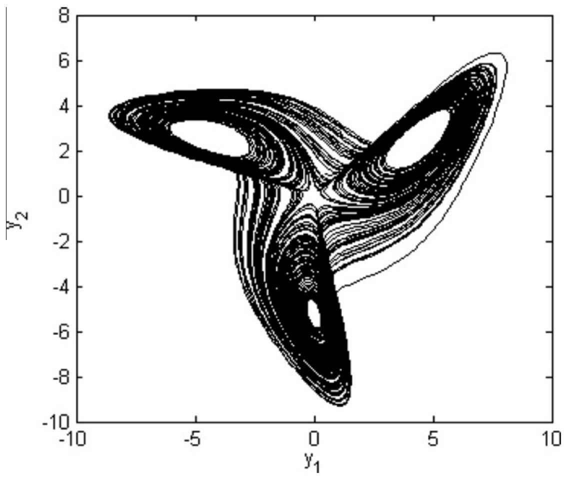

Fig. 1. Three-scroll chaotic attractor of the system (2).