Commun Nonlinear Sci Numer Simulat 20 (2015) 684–691

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Symmetry analysis of nonlinear heat and mass transfer equations under Soret effect

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ARTICLE INFO

Article history:
Received 6 April 2014
Received in revised form 20 June 2014
Accepted 21 June 2014
Available online 3 July 2014

Keywords:
Lie symmetry analysis
Group classification problem
Thermodiffusion equations
Transport coefficients
Exact solutions

ABSTRACT

Three-dimensional equations describing heat and mass transfer in fluid mixtures with variable transport coefficients are studied. Using Lie group theory the forms of unknown thermal diffusivity, diffusion and thermal diffusion coefficients are found. The symmetries of the governing equations are calculated. It is shown that cases of Lie symmetry extension arise when arbitrary elements have the power-law, logarithmic and exponential dependencies on temperature and concentration. An exact solution is constructed for the case of linear dependence of diffusion and thermodiffusion coefficients on temperature. The solution demonstrates differences in concentration distribution in comparison with the same distribution under constant transport coefficients in the governing equations.
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1. Introduction

Heat and mass transfer is one of the fundamental phenomena in diverse natural and artificial processes. Mathematical models of the phenomena have engineering applications, e.g., metal and plastic extrusion, continuous casting, glass fiber production, crystal growing. Wide using heat and mass transfer equations for modeling of different industrial processes entails the necessity to take into account some effects more accurately. That is why in recent years the mathematical models of heat and mass transfer become more complicated [1]. It is necessary to consider convective processes, complex fluid rheology, Dufort and Soret effects [2].

In order to study these complex mathematical models we need a powerful tool. It should give important information about qualitative properties of differential equations and to provide their classification and simplification. Group analysis of differential equations is one of the suitable techniques applicable to nonlinear models. This method is successfully used in application to the mathematical models of real physical processes [3,4]. Research papers [5–9] and numerous references therein articulate the current efforts in characterizing and understanding the role of symmetry approach for studying complicated mathematical models. A complete group classification for variable coefficients diffusion-convection equations is carried out in [5]. Several classes of the reaction–diffusion equations are studied in [6–8] from Lie symmetry point of view. In [9] symmetry properties of the convection model with thermodiffusion effect and constant transport coefficients are presented.

It should be noted that there are many experimental investigations of heat and mass transfer processes. The papers of experimentators from Germany and Italy [10–13] confirm that it is necessary to deal with dependence of transport coefficients on temperature and concentration of fluid mixtures. The mathematical model of related process is the system of nonlinear partial differential equations. Their analysis and exact solutions construction are important for experiments planning and identification of the dominant role of different parameters of fluid flows.

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http://dx.doi.org/10.1016/j.cnsns.2014.06.043
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The heat and mass transfer equations without convective terms with variable transfer coefficients are under consideration in the paper presented. The thermodiffusion effect is taken into account. Thermodiffusion or Soret effect refers to specific processes caused by the thermal gradient. It is a molecular transport of substance. The case of a negative thermodiffusion coefficient corresponds to positive thermodiffusion effect when the lighter component is driven towards the higher temperature region. In case of a positive thermodiffusion coefficient we have the abnormal Soret effect and the opposite situation is observed.

In Section 2 we describe the mathematical model of the process described above. In Section 3 Lie symmetry analysis of the governing equations is carried out. The determining and classifying equations are derived and solved for two cases. The first case is the thermal diffusivity dependence on temperature and concentration. The other one is constant thermal diffusivity coefficient. In Section 4 using calculated group transformations the governing partial differential equations are reduced to a system of ordinary differential equations which is treated both analytically and numerically.

2. Governing equations

It is well known that the diffusive flux of matter is given by

$$\mathbf{i} = -\rho(D\nabla C + D^{\theta}\nabla T), \tag{2.1}$$

and heat flux equals to

$$\mathbf{q} = -\kappa\nabla T. \tag{2.2}$$

In general case the liquid density ρ, coefficients of diffusion D, thermodiffusion D^{θ} and thermal conductivity κ are supposed to depend on temperature T and concentration C [15]. According to [16] and taking into account (2.1) and (2.2) we write equations of heat and mass transfer in the following form

$$\frac{\partial T}{\partial t} = \operatorname{div}(\chi\nabla T), \tag{2.3}$$

$$\frac{\partial C}{\partial t} = \operatorname{div}(D\nabla C + D^{\theta}\nabla T), \tag{2.4}$$

where t is the time, (x^1, x^2, x^3) is the coordinate vector. The thermal diffusivity coefficient $\chi = \kappa/(c_p\rho)$, the density ρ and the specific heat capacity c_p are assumed to be constant. The functions $\chi(T,C), D(T,C)$ and $D^{\theta}(T,C)$ do not vanish. The coefficients $\chi(T,C)$ and $D(T,C)$ are positive only and thermal diffusion coefficient has arbitrary sign.

Dependence of transport coefficients on temperature and concentration and the presence of the Soret effect make the governing equations new and interesting for studying. It should be noted that group analysis of equations of convective motion with variable transport coefficients are presented by the author in [14]. In this paper we continue more detailed analysis of the model when convective transfer is out of consideration.

3. Symmetry properties of the governing equations

We study group properties of the governing equations within the framework of the classical Lie approach [3]. The system contains three arbitrary elements: $\chi(T,C)$, $D(T,C)$ and $D^{\theta}(T,C)$. We carry out the group classification of Eqs. (2.3) and (2.4) with respect to these parameters. In order to find the admissible Lie symmetry group for each values of arbitrary elements we calculate the corresponding Lie symmetry algebra of infinitesimal generators. The appropriate generator for Eqs. (2.3) and (2.4) is sought out in the form

$$X = \xi^t\frac{\partial}{\partial t} + \xi^1\frac{\partial}{\partial x^1} + \xi^2\frac{\partial}{\partial x^2} + \xi^3\frac{\partial}{\partial x^3} + \eta^T\frac{\partial}{\partial T} + \eta^C\frac{\partial}{\partial C}$$

supposing that its coordinates depend on all the dependent and independent variables. The generator X satisfies the criterion of infinitesimal invariance, i.e., the action of the second prolongation $X^{(2)}$ of X on Eqs. (2.3) and (2.4) vanishes identically on the manifold of solutions of this system. So, we act by $X^{(2)}$ on (2.3) and (2.4) and then substitute into the obtained expressions the derivatives T_t and C_t implied by (2.3) and (2.4). The resulting equations can be split with respect to all derivatives of the functions T and C. As a result we get the determining and the classifying equations. After a considerable amount of calculations we find that $\xi^t = \xi^t(t)$, $\xi^i = \xi^i(t, x^1, x^2, x^3), i = 1, \dots, 3$. Then the remaining determining equations take the form

$$\begin{aligned} &\xi^2_{x^1} + \xi^1_{x^2} = 0, \qquad \xi^3_{x^1} + \xi^1_{x^3} = 0, \qquad \xi^3_{x^2} + \xi^2_{x^3} = 0,\\ &\xi^1_{x^1} - \xi^2_{x^2} = 0, \qquad \xi^1_{x^1} - \xi^3_{x^3} = 0, \qquad \xi^2_{x^2} - \xi^3_{x^3} = 0,\\ &\eta^1_{TT} = \eta^1_{TC} = \eta^1_{CC} = 0, \qquad \eta^2_{TT} = \eta^2_{TC} = \eta^2_{CC} = 0. \end{aligned} \tag{3.1}$$

The above functions are connected with the parameters of the governing equations through the equalities

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