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Loss of criticality in the avalanche statistics of sandpiles with dissipative sites

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ABSTRACT

To account for the dissipative mechanisms found in nature, non-conservative elements have been incorporated in the energy redistribution rules of sandpiles and similar models of hazard phenomena. In this work, we found that incorporating non-conservation in the form of spatially-distributed sink sites affect both the external driving and internal cascade mechanisms of the sandpile. Increasing sink densities result in the loss of critical behavior, as evidenced by the gradual evolution of the avalanche size distribution from power-law (correlated) to exponential (random). For low density cases, we found no optimal configuration that will minimize the risk of producing large avalanches. Our model is inspired by analogs in natural avalanche systems, where non-conservative elements have an inherent spatial distribution.

1. Introduction

The emergence of power-law avalanche size distributions out of simple local interactions in the sandpile model [1] spurred interest in sandpile-inspired mathematical models of natural events believed to be exhibiting self-organized criticality [2–7]. Despite their simplicity, these discrete numerical models have complemented the results from field data [2,3,7] and scaled experiments [6] in advancing our understanding of the possible mechanisms underlying natural hazards. Because the sandpile model is inspired by the avalanche dynamics observed in slowly-driven piles of sand, the insights gained from numerical implementations of these models may be used in designing effective assessment and mitigation strategies for actual landslides and avalanche events.

Needless to say, to be of practical use, several features of the model had to be adjusted to incorporate realistic scenarios. Apart from using continuous "energy" values [8], asymmetric redistribution rules [4,5] that may be time-varying [6], and non-uniform driving magnitudes [7], several models incorporate non-conservative rules for the distribution of energies during individual toppling events [4,5,9]. Non-conservation, which is prevalent in many open systems in the natural setting, is oftentimes introduced as a global parameter $C \in (0, 1]$ describing the fraction of energy transferred to the site's neighbors during local toppling events. For sandpiles with cell states that have continuous values, the introduction of non-conservative rules resulted in stretched exponential distributions, signifying the loss of critical behavior [9]. As applied to landslide modeling, the level of non-conservation explained the origin of key features of the empirical landslide distributions, particularly the power-law exponents and the occurrence of systematic deviations for small landslides [5].

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The use of a constant non-conservation parameter is based on the assumption that all local regions will behave similarly at all times. In the natural setting, however, dissipative elements have an inherent spatial distribution. For example, in actual surficial landslides, the downward flow of material along an incline is oftentimes affected by physical structures in the form of natural obstacles (boulders, trees and root networks, or channels created by water flow) [10] and man-made reinforcement structures [11] that decrease the extent of the avalanche process. The interplay between the (1) spatial extent and (2) distribution of these obstructions is therefore of paramount importance in determining the overall susceptibility of a sloped region to an avalanche event.

Inspired by the parallels with actual landslides in nature, we investigate the effect of non-global, spatially-distributed non-conservative elements in the sandpile grid, the paradigm model of self-organized criticality. Certain sites in the sandpile grid are made to be *sinks* that completely absorb and dissipate the energy they received during local toppling events. Previous works have explored the effect of sink configurations in the pattern formation of Abelian sandpiles [12]. Here, we consider the Zhang sandpile, i.e. having continuous-valued states [8] and focus on the effect of the sinks in the statistical distributions of avalanche sizes. From a modeling point of view, the results presented in this work are important because they demonstrate the loss of critical behavior in the statistical signatures of the system due to the presence of dissipative elements. Moreover, we observe that the occurrence of large avalanches is more influenced by density rather than the arrangement of sinks; in particular, our analyses involving low-density cases reveal that there is no optimal configuration that significantly reduces the risk of large avalanches. This observation, which may have an analog in the natural setting, should be accounted for in designing mitigation strategies for actual avalanches.

2. Model

2.1. Sandpile model formalism

The sandpile model utilizes discrete space and time to model the internal cascading mechanisms present in many self-organised critical systems [1]. Formally, the sandpile model is a cellular automaton (CA) defined by the four-tuple $(G = L^2, N, E, \delta, f : t \rightarrow t+1)$, with the following components:

- $G = L^2$ is the two-dimensional grid of cells (x, y) arranged in a square lattice configuration;
- N is the neighbourhood describing the extent of interaction of the cells;
- E is the set of all possible states or "energy" values of a given cell site;
- δ is the external trigger added to random locations every discrete time step; and
- $f : t \rightarrow t+1$ is the set of rules that map how the grid evolves in discrete time (the specific rules are to be described later).

In this case, we use the following values for our first four parameters: $L = 64$ such that $G = 64^2$ cells; N is the von Neumann neighbourhood, i.e. for any cell (x, y), the neighborhood $n(x, y) = \{(x-1, y),\ (x+1, y),\ (x, y-1),\ (x, y+1)\}$; E is the set of continuous values $E = [0, E_{thres})$, where $E_{thres} = 1.0$; and δ is a constant that is set to be several orders of magnitude less than $E_{thres},\ \delta = 5 \times 10^{-3}$. The use of continuous energy states E is patterned after Zhang [8] while the value of the external trigger δ used is guided by previous large-grid implementations of Lübeck [13].

We note that for ease of implementation, we inactivate the edges of the grid, i.e., $e(x, y) \rightarrow 0$ if $x = 1,\ x = L,\ y = 1$, or $y = L$. This ensures that all the other cells will have a von Neumann neighbourhood N for use by the mapping rules f.

The mapping $f : t \rightarrow t+1$ guides the evolution of the grid G in discrete time. For the sandpile model, the mapping f is composed of two rules for the external triggering and the internal energy redistribution in the grid. The former is continuously applied every time step while the latter is subject to the neighbourhood configurations at any particular time.

Every discrete time t, the external triggering part of the mapping f triggers a random site (x, y) by adding δ to its current energy state $e(x, y)^t$,

$$e(x, y)^{t+1} \rightarrow e(x, y)^t + \delta \tag{1}$$

After the external triggering rule, the internal energy redistribution rule of the mapping f is applied by checking the triggered site for "stability." When the triggered site has $e(x, y) \geqslant E_{thres}$, it is considered to be critical, as it matches or exceed the threshold value E_{thres}. When the grid contains one or more critical cells, time is frozen (i.e. no new δ is added) and the following rule is applied until all the cells have energies that are below E_{thres}:

$$e(n) \rightarrow e(n) + \frac{e(x, y)}{4} \tag{2}$$

where $e(n)$ is the energy of each of the cells in the neighbourhood N, $\{e(x-1, y),\ e(x+1, y),\ e(x, y-1),\ e(x, y+1)\}$. Needless to say, the redistribution rule results in the zeroing out of the energy of the cell in consideration and the corresponding increase in the energy states of the four nearest neighbours. This can cause nearest neighbor cells themselves to be critical, resulting in a cascade event wherein repeated implementations of the redistribution rule Eq. (2) is applied. The number of regular cells that participated in the cascade is recorded as the avalanche size A.