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Asymptotic behavior of a stochastic non-autonomous predator–prey model with impulsive perturbations

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ABSTRACT

This paper is concerned with a stochastic non-autonomous Lotka–Volterra predator–prey model with impulsive effects. The asymptotic properties are examined. Sufficient conditions for persistence and extinction are obtained, our results demonstrate that the impulse has important effects on the persistence and extinction of the species. We also show that the solution is stochastically ultimate bounded under some conditions. Finally, several simulation figures are introduced to confirm our main results.

1. Introduction

The stochastic non-autonomous Lotka–Volterra predator–prey model with white noise is expressed by

$$\begin{cases} \mathrm{d}x_1(t) = x_1(t)[r_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t)]\mathrm{d}t + \sigma_1(t)x_1(t)\mathrm{d}B_1(t), \\ \mathrm{d}x_2(t) = x_2(t)[-r_2(t) + a_{21}(t)x_1(t) - a_{22}(t)x_2(t)]\mathrm{d}t + \sigma_2(t)x_2(t)\mathrm{d}B_2(t). \end{cases} \tag{1}$$

The stochastic processes $x_1(t)$ and $x_2(t)$ represent, respectively, the prey and predator populations. And $r_i(t)$ denotes the intrinsic growth rate of the corresponding population at time $t, a_{11}(t)$ and $a_{22}(t)$ represent the density-dependent coefficients of the prey and the predator, respectively. The coefficient $a_{12}(t)$ is the capturing rate of the predator and $a_{21}(t)$ stands for the rate of conversion of nutrients into the reproduction of the predator. Both $\sigma_1(t)$ and $\sigma_2(t)$ are the coefficients of the effects of environmental stochastic perturbations on the prey and predator population, respectively. The standard Brownian motions $B_i(t),\quad i = 1,2$, are independent each other. The functions $r_i(t), a_{ij}(t)$ and $\sigma_i(t)$ $(i,j = 1,2)$ are continuous bounded functions on $\mathbb{R}_+ = [0,+\infty)$.

The stochastic model (1) has received great attention and many good results have been published, see [1–5] and the references cited therein. Liu and Wang analyzed the extinction and persistence of species of model (1) in [1]. In [2,3] Rudnicki investigated system (1) in the autonomous case, showed that the distributions of the solutions are absolutely continuous and proved that the densities can converge in L^1 to an invariant density or can converge weakly to a singular measure. There are many other papers in the literature on models with white noise, the readers can refer to [6–12] and the references cited therein.

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However, in the real world, due to some natural and man-made factors, the growth of species often suffers from some discrete changes of relatively short time interval at some fixed times, such as drought, flooding, hunting, planting, etc. These phenomena cannot be considered continually, so in this case, system (1) cannot capture these phenomena. Introducing the impulsive effects into the model may be a reasonable way to accommodate such phenomena, see [13,14].

Lots of deterministic population dynamical systems with impulsive effects have been proposed and studied. Many results on dynamical behavior for such systems have been reported, see e.g. [15–18] and the references therein. Recently, authors of [19–22] considered the stability of stochastic differential equation (SDE) with impulsive effects. However, so far as we know, there are few papers published which study the impulsive stochastic population model, see [23–25]. By now, there are no results related to the stochastic predator–prey system with impulsive effects.

Inspired by the above discussions, we propose the following stochastic Lotka–Volterra predator–prey model with impulsive effects:

$$\begin{cases} dx_1(t) = x_1(t)[r_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t)]dt + \sigma_1(t)x_1(t)dB_1(t), \quad t \neq t_k,\ k \in N, \\ dx_2(t) = x_2(t)[-r_2(t) + a_{21}(t)x_1(t) - a_{22}(t)x_2(t)]dt + \sigma_2(t)x_2(t)dB_2(t), \quad t \neq t_k,\ k \in N, \\ x_1(t_k^+) - x_1(t_k) = b_{1k}x_1(t_k), \quad k \in N, \\ x_2(t_k^+) - x_2(t_k) = b_{2k}x_2(t_k), \quad k \in N, \end{cases} \tag{2}$$

where N denotes the set of positive integers, $0 < t_1 < t_2 < \cdots, \lim_{k\to+\infty} t_k = +\infty$. For biological meanings, we impose the following restriction on b_{ik}:

$$1 + b_{ik} > 0, \quad i = 1,2,\ k \in N.$$

For example, if $b_{ik} > 0$, the impulsive effects may denote planting of the species, while $b_{ik} < 0$ may represent harvesting.

The main aims of this paper are to investigate how impulses affect the asymptotic behaviors of system (2). The rest of this paper is organized as follows. In Section 2, we show that the solution of system (2) is global and positive. In Section 3, we study the conditions under which it is observed extinction or persistence of the species modeled by system (2). The stochastically ultimate boundedness is examined in Section 4. Finally, we present several numerical simulations to illustrate out results and the effects of impulse.

2. Global positive solutions

For convenience, we adopt the following notations. If $f(t)$ is a continuous bounded function on $\mathbb{R}_+$, define $f^u = \sup_{t\in\mathbb{R}_+} f(t), f^l = \inf_{t\in\mathbb{R}_+} f(t)$. $\overline{f(t)} = t^{-1}\int_0^t f(s)ds, f^* = \limsup_{t\to+\infty} f(t), f_* = \liminf_{t\to+\infty} f(t)$. And $\prod_{i=1,2} y(i)$ denotes $\prod_{i=1,2} y(i) = y(1)y(2)$. Throughout this paper, we assume that $a_{ii}^l > 0, a_{ij}^l \geqslant 0, r_2^l > 0, i \neq j, \quad i,j = 1,2$. Moreover, we always assume that a product equals unity if the number of factors is zero.

Before we discuss the asymptotic properties of solutions to (2), we should first guarantee the existence of global positive solutions. First, we give the definition of solutions to impulsive stochastic differential equations (ISDE).

Definition 1 [23]. For a given ISDE:

$$\begin{cases} dX(t) = F(t,X(t))dt + G(t,X(t))dB(t), \quad t \neq t_k,\ k \in N, \\ X(t_k^+) - X(t_k) = B_kX(t_k), \quad k \in N, \end{cases} \tag{3}$$

with the initial condition $X(0)$. A stochastic process $X(t) = (X_1(t), X_2(t), \ldots, X_n(t))^T$ is said to be a solution of (3) on $\mathbb{R}_+$, if the following conditions are satisfied:

(a) $X(t)$ is $\mathfrak{F}_t$-adapted and is continuous on $(0,t_1)$ and each interval $(t_k, t_{k+1}), k \in N; F(t,X(t)) \in \mathcal{L}^1(\mathbb{R}_+;\mathbb{R}^n)$, $G(t,X(t)) \in \mathcal{L}^2(\mathbb{R}_+;\mathbb{R}^n)$, where $\mathcal{L}^k(\mathbb{R}_+;\mathbb{R}^n)$ is all $\mathbb{R}^n$-valued measurable $\mathfrak{F}_t$-adapted processes $f(t)$ satisfying $\int_0^T |f(t)|^k dt < \infty$ almost surely for every $T > 0$.
(b) For each $t_k, k \in N, X(t_k^+) = \lim_{t\to t_k^+} X(t)$ and $X(t_k^-) = \lim_{t\to t_k^-} X(t)$ exist and $X(t_k) = X(t_k^-)$ with probability one.
(c) $X(t)$ obeys the equivalent integral equation of (3) for almost every $t \in \mathbb{R}_+ \setminus t_k$ and satisfies the impulsive conditions at each $t = t_k, \quad k \in N$, with probability one.

We are now in position to prove the existence of the positive solution to stochastic system (2) with impulsive effects, the following theorem states this.

Theorem 1. *For any given initial value* $(x_{10}, x_{20}) \in \mathbb{R}_+^2 = \{(x,y) \in \mathbb{R}^2 | x > 0, y > 0\}$, *the system* (2) *has a unique solution* $x(t) = (x_1(t), x_2(t))$ *on* $t \geqslant 0$ *and the solution will remain in* $\mathbb{R}_+^2$ *a.s. (almost surely).*

Proof. We follow [23]. Consider the following SDE without impulse: