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Optimal control for a tuberculosis model with undetected cases in Cameroon

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ABSTRACT

This paper considers the optimal control of tuberculosis through education, diagnosis campaign and chemoprophylaxis of latently infected. A mathematical model which includes important components such as undiagnosed infectious, diagnosed infectious, latently infected and lost-sight infectious is formulated. The model combines a frequency dependent and a density dependent force of infection for TB transmission. Through optimal control theory and numerical simulations, a cost-effective balance of two different intervention methods is obtained. Seeking to minimize the amount of money the government spends when tuberculosis remain endemic in the Cameroonian population, Pontryagin's maximum principle is used to characterize the optimal control. The optimality system is derived and solved numerically using the forward–backward sweep method (FBSM). Results provide a framework for designing cost-effective strategies for diseases with multiple intervention methods. It comes out that combining chemoprophylaxis and education, the burden of TB can be reduced by 80% in 10 years.

1. Introduction

Tuberculosis (TB) is a preventable and curable disease caused by *Mycobacterium tuberculosis* (Mtb) that most often affects the lungs. To date, TB claims the second largest number of victims due to a single infectious agent right after Human Immunodeficiency Virus and Acquired Immune Deficiency Syndrome (HIV/AIDS) [39]. According to the WHO data published in April 2011 the TB case detection rate (all forms) in Cameroon was last reported at 69% in 2010 and the TB deaths reached 3647 or 1.54% of total deaths. The age adjusted death rate of 21.89 per 100,000 of population ranks Cameroon 68th in the world [39].

Adult mortality has a significant effect on national economies, through both the direct loss of productivity among those of working age and by altering fertility, incentives for risk-taking behavior, and investment in human and physical capital [24]. TB is the most important cause of adult death due to infectious disease after HIV/AIDS. TB has its greatest impact on adults between the ages of 15 and 59 [38]. Therefore most economically productive persons in society, parents on whom development and survival of children depend, are affected. TB places an extraordinary burden on those afflicted by the disease, their families, communities and on government budgets. In fact, the greatest burden of TB falls on productive adults who, once

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infected, are weakened and often unable to work. The burden of taking care of sick individuals usually falls to other family members and, besides putting them at greater risk of infection, can lower their productivity [24]. Diagnosed individuals with TB are often medically quarantined for a period of time, which can affect their financial well-being. The infected population has an economic impact on their families and in turn their countries' national economies through their inability to contribute financially, as they are often unable to be productive workers. Along with loss of productivity, the TB treatment charge can be for the Government significant. Average household spending on TB can account for as much as 8–20% of annual household income, varying by region [13,35].

Some TB models in the literature have discussed control of the disease by looking at the role of disease transmission parameters for time dependent control strategies. Time dependent control strategies have been applied for the studies of HIV models [15,22], two strain tuberculosis models [21], a TB model with lost sight class to reduce the rate by which people become lost sight [14], and a SARS model with quarantine [41]. Some authors [19,33] discussed the optimal control on a model with reinfection. Time dependent optimal controls on chemoprophylaxis of both diagnosed and latently infected individual, and disease relapses are considered in [1] to reduce the actively infected individual populations. In their recent article, Silva and Torres [37] studied the time optimal control of TB in Angola using a model with early and persistent latency. Approaches of studying control strategies produced valuable theoretical results which suggest or design epidemic control programs. However, none of the previous studies considered the key combined role of undiagnosed infectious population and quitting of the treatment on TB propagation. The goal in this paper is to minimize the number of people who are undiagnosed infectious, lost sight, thus also the number of people who die due to the infection. The number of TB new-cases, even if the success rate of treatment remains the same will decrease implicitly. In this paper, we compute an optimal way to minimize the cost of TB on a calibrated model taking into account the Cameroonian last trend of the diseases and resulting parameters. The TB cost here means the expenses to fight TB and save human life.

Two control strategies are considered. The first consists of education of the population about TB and large scale diagnosis campaigns. Through education, one can decrease the number of undiagnosed infectious people inside the population. Using the combined effect of education and free of charge treatment, one can first reduce the number of people infected in the population, make return people who have quitted the treatment to the hospital and accelerate the detection of newly infected people who would stay to be treated naturally or through self-medication and traditional medicine. A proportion of them will die without appropriate treatment. According to FAO, less than 73% of the Cameroonian population goes to the hospital after first disease symptoms. Because of the inaccessibility of certain regions of the country, access to health facilities is often difficult. Besides treatment facilities, some rural and even urban population prefer sometimes to use traditional medicine or self-medication for which the efficiency has not been established yet for TB. An increase in the treatment access should help to reduce the lost sight and undiagnosed classes. The immediate consequence will be a reduction in the number of infectious and then, on the number of diagnosed infectious.

The second TB control approach is the chemoprophylaxis treatment. According to the National Committee of Fight against TB (NCFT), the chemoprophylaxis of latently infected population is currently not practiced in Cameroon. The number of latently infected individuals that may develop an active TB will decrease if the chemoprophylaxis is practiced. The impact of both strategies on TB dynamics will be discussed. We intend to determine optimal control strategies that minimize not only undiagnosed infectious but also lost-sight and diagnosed infectious individuals which are the source of TB spread. The optimal controls are completely characterized and optimal solutions are depicted.

2. Modeling intervention methods

Mycobacterium tuberculosis (Mtb) spreads through the air from an infectious person to a susceptible person. Two kinds of tests are used to determine if a person has been infected with TB bacteria: the tuberculin skin test and TB blood analysis. A positive TB skin test or TB blood test only tells that a person has been infected with TB bacteria and does not tell whether the person has latent TB infection or has progressed to TB disease [12]. Other tests, such as a chest X-ray and a sample of sputum, are needed to see whether the person has TB disease. Some TB treatment and prevention options are carried out in some rural and urban hospitals in Cameroon. However, the treatment of mild infections, classified as latent infections in our model, is not effective in Cameroon. On the other hand, infective individuals classed as infectious in our model, require a hard treatment of six months in the hospital. As preventive measures, population can be diagnosed, and latent TB infections can be treated to reduce the bacterial load in their body. This last approach effectively reduces the risk that TB infection will progress to TB disease. Certain groups are at very high risk of developing TB disease once infected. Every effort should be made to begin appropriate treatment and to ensure completion of the entire course of treatment for latent TB infection [12].

A finite (non-constant) total population at time t, denoted by $N(t)$, is sub-divided into the following mutually exclusive sub-populations:

- S susceptible: healthy people not yet exposed to TB;
- E latently infected: exposed to TB but not infectious;
- I diagnosed infectious: have active TB confirmed after a sputum examination in a hospital;
- J undiagnosed infectious: have not yet been to a hospital for diagnosis but are active for confirmation by a sputum examination;