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Stabilizing the unstable periodic orbits of a hybrid chaotic system using optimal control

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ABSTRACT

In this paper, we are interested in the control of a chaotic hybrid system with an application to Chua's system. It is known that chaotic attractors contain an infinite number of unstable periodic orbits (UPO) with different lengths, our idea consists in stabilizing a predetermined orbit of a given length by using an optimal control method. Our approach is to determine the switching instants from one subsystem to the other while minimizing the difference between two successive orbits. Should the switchings be state dependent, as is the case for the well known Chua's circuit, then our approach consists in perturbing the switching boundaries such that the system trajectory hits those boundaries at the specified instants. Numerical simulations illustrating the efficiency of the proposed method are presented.

1. Introduction

The last two decades have witnessed a great interest in nonlinear dynamical systems with a special attention to those having chaotic behavior. In fact, the emergence of new mathematical and numerical tools played a crucial role to understand, characterize and quantify chaos. This helped researchers to identify chaos in many scientific disciplines such as biology [1], chemistry [2] and engineering applications [3–5]. Actually, the research on chaotic systems can be classified into three main streams: Investigating new chaotic systems [6,7], synchronizing chaotic systems[8,9] and controlling chaos [10]. Our present work falls within the third stream.

After the pioneering work on controlling chaos introduced by Ott et al. [11], there have been many other attempts to control chaotic systems with three main aims. The first, which is merely classic, consists in stabilizing one of the unstable equilibrium points [12–15]. The second, uses the control strategy to achieve synchronization [16–19] or anti-synchronization [20,21]. The third, is the most important when it comes to the control of chaotic systems, and concerns the stabilization of unstable periodic orbits embedded in the chaotic attractor [22–27].

As a matter of fact, the stabilization of chaotic systems UPO's is of particular interest. Indeed, the need to stabilize a UPO of a chaotic system raises from the application itself. For instance, the voltage ripple in an H-bridge inverter is much smaller when it is behaving periodically [28]. When a stepper motor behaves chaotically [29], it becomes impossible to use it in an

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open loop control system. A chaotic macroeconomic growth cycle [30] is unpredictable and leads to difficulty in organizing long delay projects. The aforementioned examples indicate that rendering a chaotic system periodic is a worthy endeavor.

One of the most appealing method to control UPO's is the time-delayed feedback control (TDFC) that has been described by Pyragas [22]. It consists in adding an external force proportional to the error between the current state of the system and the one period delayed state. The continuous TDFC has been successfully applied to control the UPO's of Chua's system [26].

In this work, we are interested in hybrid chaotic systems characterized by non-smooth dynamic systems [31]. In fact such systems are well known in power electronics such as the H-bridge inverter [32,33] and DC/DC power converters [5]. Hybrid systems are known to toggle among several continuous-time dynamical systems according to a discrete switching law. Should the continuous-time systems be unstable and the global behavior be bounded, then such systems are more likely to exhibit chaotic motion due to the existence of stretching and folding reasons [34,35]. To control chaos in hybrid systems several methods have been developed such as the time-delayed impulsive control [36], discrete state feedback [37] and pole assigning [38].

In the present paper, we suggest to control a UPO of a hybrid chaotic system by using an optimal controller that intends to optimize the switching instants by optimizing a performance criteria in terms of the error between the orbit and the delayed one. If the switching of the hybrid system is rather state dependent, then our approach consists in modifying the switching boundary such that the system trajectory hits the boundary at the specified time. The boundary perturbation should be annihilated when the predetermined UPO is stabilized. To illustrate the efficiency of our method we consider Chua's system as a switched system that toggles among three linear systems according to the state position in the phase space. An in depth analysis of the periodic and chaotic behaviors in the Chua circuit system viewed as a switching system can be found in [39–41].

This paper is organized as follows. In the second section we formulate the problem with respect to a general hybrid system. In the third section we apply the proposed control method to the well known Chua's system and we illustrate its efficiency using numerical simulations. Finally in the last section we present some concluding remarks.

2. Problem formulation

A hybrid dynamic system is an interaction between continuous-time systems described by differential equations and discrete-event dynamics. Such systems can be described as follows:

$$\dot{x}(t) = f(x(t), u(t), q(t)) = f_{q(t)}(x(t), u(t)), \quad x(t_0) = x_0 \tag{1}$$

$$q(t) = \delta(x(t), q(t^-)), \quad q(t_0) = q_0 \tag{2}$$

where $u(t) \in \mathbb{R}^{n_u}$ is an input signal to the hybrid system, $q(t) \in Q = \{n_1, \ldots, n_{|Q|}\}$ (here $|Q|$ is the cardinal of Q) is the discrete state that defines the active subsystem $f_{q(t)}$ and $x(t) \in \bigcup_{i \in Q} X_i \subseteq \mathbb{R}^n$ is the continuous state vector; X_i's are closed subsets (possibly unbounded) with pairwise disjoint interiors. We denote by $q(t^-)$ the left limit of the discrete state at time t, we use $(x(t), q(t))$ to denote the hybrid state belonging to the hybrid space $H = \mathbb{R}^n \times Q$ and $\delta : H \rightarrow Q$ stands for the discrete transition map. The transition between the hybrid state $(x(t^-), q(t^-))$ to $(x(t), q(t))$ occurs along a switching surface $\sigma_{ij} = \{x \in X_i \cap X_j\}$ with $q(t^-) = i$ and $q(t) = j$. It is also assumed that $x(t^-) = x(t)$ so $x(t)$ is piecewise continuous $\mathcal{C}^1$ function for all $t \geqslant 0$. We say $x(t)$ is a trajectory of (1) if for every $t \geqslant 0$ such that $\dot{x}(t)$ is defined, the Eq. (1) holds for all i with $x(t) \in X_i$.

Let us furthermore assume that within a finite time interval of length T ($t \in [t_0, t_0 + T]$), the trajectory $x(t)$ undergoes a finite number of switchings N at times $\{\tau_0, \tau_1, \ldots, \tau_{N+1}\}$; where $\tau_0 = t_0$ and $\tau_{N+1} = t_0 + T$. Therefore we can define a sequence $S = \{s_0, s_1, \ldots, s_N\}$ of discrete states associated with the continuous trajectory $x(t)$ such that $s_m \in Q$ and

$$s_m = q(t), \quad \tau_m \leqslant t < \tau_{m+1}, \ldots m = 0, 1, \ldots, N \tag{3}$$

The sequence S together with the trajectory $x(t)$ define a hybrid trajectory for which we define the periodicity property as given in [36] and that we recall hereafter.

Definition 1. For any $t \geqslant 0$, if there exists some constant $T > 0$ such that $x(t) = x(t + T)$, then we say the system is traveling on a continuous T-periodic orbit.

Definition 2. For any integer $m > 0$, if there exists some integer constant $k > 0$ such that the symbolic sequence has the property $s_m = s_{m+k}$, then we say the system is traveling on a discrete period k orbit.

It is worth noting that a trajectory with a periodic sequence S does not imply that the continuous trajectory is periodic. However, if both definitions are satisfied, then we say the system is traveling on a hybrid periodic orbit.

In this work, we focus on autonomous hybrid systems having a chaotic behavior. Knowing that a chaotic attractor contains an infinite number of UPO's with different lengths, then a predetermined UPO of length T_p denoted by $x_p(t)$ should satisfy (1), that is we have:

$$\dot{x}_p(t) = f(x_p(t), q_p(t)), \tag{4}$$