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On the dynamics of bistable micro/nano resonators: Analytical solution and nonlinear behavior

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ABSTRACT

With the rapid development of micro/nano-electro-mechanical systems (MEMS/NEMS), arch shaped resonators are becoming increasingly attractive for different applications. Nevertheless, the dynamics of bistable resonators is poorly understood, and the conditions for their appropriate performance are not well known. In this paper, an initially curved arch shaped MEMS resonator under combined DC and AC distributed electrostatic actuation is investigated. A reduced order model obtained from first mode Galerkin's decomposition method is used for numerical and analytical investigations. We have used the Homotopy Analysis Method (HAM) in order to derive analytical solutions both for the amplitude and the temporal average of nonlinear vibrations. The obtained analytical expressions, validated by numerical simulations, are able to capture nonlinear behaviors including softening type vibrations and dynamic snap-through. We have used the derived analytical results in order to study the nonlinear vibrations of the bistable MEMS resonator. According to our results, in the bistability region the overall dynamic response of the system is described by means of a pair of softening type frequency responses merging together in a specific frequency band. The dynamic snap-through is then described by transitions between these two frequency responses, each of which corresponding to one of the stable configurations of the arch. This fresh insight to the problem can be used in the design and optimization of bistable resonators and determination of their sharp roll-off frequencies. A feature that can be implemented in the design of bandpass filters.

1. Introduction

Micro-electro-mechanical systems (MEMS) resonators have gained tremendous attention due to their various applications such as accelerometers [1], load cells [2], gyroscopes [3], gas and chemical sensors [4] and biosensors [5]. A list of advantages including low power consumption, low cost and improved reliability makes MEMS resonators suitable for the aforementioned applications. So far, most of the presented MEMS resonators are comprised of a fixed electrode and an

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electrostatically actuated movable electrode. Dynamic behaviors of these types of resonators are widely investigated in the literature [6–11]. These nonlinear behaviors include hardening or softening effects, dynamic pull-in or chaotic vibrations.

In recent years, bistable MEMS devices have gained lots of attention in the research community. These interests are motivated by applications of bistable MEMS devices including microrelays [12], microvalves [13], switches [14] and MEMS based memories [15]. Bistability refers to the ability of the system to operate in more than one stable configuration at certain values of geometrical and loading parameters. A MEMS device composed of an arch shaped microbeam is a typical representative of the family of bistable MEMS devices. These bistable microbeams are intentionally made in a curved (arch) configuration. This can be achieved by either micro machining or buckling a straight beam under compressive axial loads [16,17]. In addition to other nonlinear phenomena, these mechanically bistable MEMS undergo a nonlinear phenomenon, known as snap-through, which is responsible for the precipitate mechanical transitions of the micro arch between its convex and concave configurations. Various drive mechanisms including electromagnetic [18], mechanical touch [19], electrostatic [20], thermal [21] and optical [22] were reported as the starters of the so-called snap-through motion for bistable arch shaped MEMS devices.

Many researchers studied the static snap-through and pull-in response in electrostatically actuated bistable MEMS devices. Analytical and experimental studies on the static and dynamics of arch shaped MEMS are reported in the works of Krylov et al. [20,23–25]. Their reports include: (i) derivation of the governing equations of motion based on the Euler–Bernoulli beam theory and the Galerkin's method, (ii) presentation of bistability conditions and (iii) investigation of the effects of various parameters. They also studied response of the system to DC electrostatic shocks using phase plane analysis. Zhang et al. [26] experimentally studied the bistability of an arch shaped micro-machined beam. They construed snap-through, due to its zero power consumption and precipitate motion, as an appropriate phenomenon in the development of highly sensitive sensors. In order to study the pull-in and snap-through instabilities, finite and boundary element methods were used by Das and Batra [27].

A few research groups have focused on the resonant behaviors of the bistable arch shaped microbeams. Casals-Terre et al. [28] demonstrated an initially curved double clamped micro beam under combined DC and AC electrostatic actuation. They studied, mainly due to snap-through triggered by mechanical resonance, the possibility of transitions between stable configurations. Ouakad and Younis [17] investigated an arch shaped MEMS resonator. They scrutinized the dynamic behavior of the resonator using Galerkin's method. Also, they used a perturbation technique (the Multiple Scales Method) in order to examine the response of the resonator under small DC and AC actuations. The main limitation of the perturbation method is that it is only valid for small deflections of the resonator [29]. Thus, only problems with weak nonlinearities can be analyzed by this approach. This assumption is not valid for large displacements imposed by dynamic snap-through. In another work, Younis et al. [16] studied an arch shaped MEMS resonator; they showed, both numerically and experimentally, various interesting nonlinear phenomena including softening behavior, dynamic snap-through as well as dynamic pull-in. They also suggested application of these types of MEMS as bandpass filters or low-powered switches. Recently, they [30,31] have reported further discussions on the bistable-MEMS-based bandpass filters.

Unlike perturbation techniques, the Homotopy Analysis Method (HAM), originally introduced and developed by Liao [32–34], does not depend on any small parameter assumption. This method provides a powerful straightforward tool for representation of the solution of highly nonlinear equations in the form of convergent series. Also, unlike other techniques targeting a series solution, the HAM can guarantee convergence of the series solution by introducing a so-called convergence-control parameter [35,36]. Because of its effectiveness and simplicity, HAM has been widely implemented in the literature for analysis of various highly nonlinear problems in recent years [37–40].

Though significant progresses in the investigation of the dynamic behaviors of the arch shape microbeam, there are still many issues under discussion which are beyond the capability of classical perturbation techniques. During analysis of curved microbeam behaviors, the highly nonlinear frequency responses elucidate the necessity of using a precise analytical solution. HAM method, with its strong capability, has a great potential to relate the microbeam response with the input voltages. This paper aims to study the nonlinear behavior of an electrostatically actuated bistable MEMS resonator using the HAM method. We assume that a double clamped micro arch under combined static DC and harmonic AC electrostatic actuation acts as a resonator near its first fundamental frequency. As a novel approach and with the aim of capturing dynamic snap-through instabilities, we have described the dynamic displacement of the arch by two unknowns, namely the amplitude and the temporal average of nonlinear vibrations. The latter directly describes the snap-through behavior of the arch. We present analytical expressions for these two unknowns providing appropriately precise tools for description of the nonlinear behaviors of the bistable MEMS resonator.

The rest of the paper is organized as follows. In Section 2, a first mode Galerkin's decomposition method is used to convert the corresponding Euler–Bernoulli based governing partial differential equation to a second order nonlinear differential equation. In Section 3, we use HAM in order to obtain analytical solution for the nonlinear vibrations of the arch resonator. Section 4 continues with the verification of the obtained analytical solutions via numerical simulations. Also, we investigate in detail a typical case of nonlinear vibrations of the presented bistable resonator using both analytical solutions and numerical simulations. Then, we present a novel interesting description for the bistable vibrations and consequent snap-through instability of the arch resonator. We show that the nonlinear snap-through jumps can be described by merging of two frequency responses in the bistability region, each of which corresponding to one of the stable configurations of the arch. Further conclusions and discussions are presented in Section 5, where we conclude that this work is a forward step in the