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Cnoidal and snoidal wave solutions to coupled nonlinear wave equations by the extended Jacobi's elliptic function method

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ABSTRACT

This paper studies two nonlinear coupled evolution equations. They are the Zakharov equation and the Davey–Stewartson equation. These equations are studied by the aid of Jacobi's elliptic function expansion method and exact periodic solutions are extracted. In addition, the Zakharov equation with power law nonlinearity is solved by traveling wave hypothesis.

1. Introduction

The study of nonlinear evolution equations (NLEEs) appears everywhere in Applied Mathematics and Theoretical Physics including Engineering Sciences and Biological Sciences. These NLEEs play a key role in describing key scientific phenomena. For example, the nonlinear Schrödinger's equation describes the dynamics of propagation of solitons through optical fibers. The Korteweg–de Vries equation models the shallow water wave dynamics near ocean shore and beaches. Additionally, the Schrödinger–Hirota equation describes the dispersive soliton propagation through optical fibers. These are just a few examples in the whole wide world of NLEEs and its applications [1–32].

While the above mentioned NLEEs are scalar NLEEs, there is a large number of NLEEs that are coupled. Some of them are two-coupled NLEEs such as the Gear–Grimshaw equation [30], while there are several others that are three-coupled NLEEs. An example of a three-coupled NLEE is the Wu–Zhang equation [32]. These coupled NLEEs are also studied in various areas of Theoretical Physics as well. In this paper there will be two two-coupled NLEEs that will be studied. They are the Zakharov equation (ZE) and the Davey–Stewartson equation. The first equation appears in the study of Plasmas, while the second equation shows up in the study of wave packet in water waves of finite depth.

The integrability aspects of these two equation will be the main focus in this paper. There will be a couple of integration techniques that will be applied to solve these equations. They are the traveling wave hypothesis as well as the extended Jacobi's elliptic function method. The first method will be applied to find soliton solutions to the Zakharov equation and in this context the power law nonlinearity will be taken into account. The second method will be applied to extract cnoidal and snoidal waves for both the equations in this paper without power law nonlinearity.

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2. Governing equations

The first one, we studied in this paper is the generalized-Zakharov equations for the complex envelope $u(x,t)$ of the high-frequency wave and the real low-frequency field $v(x,t)$, which takes the form

$$\begin{aligned} &ihu_t + u_{xx} - 2\lambda|u|^2u + 2uv = 0, \\ &v_{tt} - v_{xx} + (|u|^2)_{xx} = 0, \end{aligned} \tag{2.1}$$

where the cubic term in Eq. (2.1) describes the nonlinear-self interaction in the high frequency subsystem, such a term corresponds to a self-focusing effect in plasma physics. The coefficient λ is a real constant that can be a positive or negative number [24].

The second one, we studied in this paper is (2 + 1)-dimensional Davey–Stewartson equation. The two-dimensional Davey–Stewartson equation reads

$$\begin{aligned} &iu_t + u_{xx} - u_{yy} - 2|u|^2u - 2uv = 0, \\ &v_{xx} + v_{yy} + 2\left(|u|^2\right)_{xx} = 0. \end{aligned} \tag{2.2}$$

This equation is completely integrable and often used to describe the long time evolution of a two-dimensional wave packet [24,25].

In this paper, we will use the extended Jacobian elliptic function expansion method to construct new exact periodic solutions of the generalized-Zakharov and coupled Davey–Stewartson equations. Under limiting conditions, the periodic solutions can be reduced to the corresponding solitary wave solutions.

3. Traveling wave hypothesis

The Zakharov equation with power law nonlinearity will be studied in this section. The dimensionless form of the ZE, with power law nonlinearity, that is going to be studied in this paper is given by

$$iq_t + aq_{xx} + b|q|^{2n}q = cqr \tag{3.1}$$

$$r_{tt} - k^2r_{xx} = \alpha\left(|q|^{2n}\right)_{xx} \tag{3.2}$$

Eqs. (3.1) and (3.2) are the ZE with power law nonlinearity and the parameter n is the power law nonlinearity parameter. The dependent variable $q(x,t)$ is the complex valued function while $r(x,t)$ is a real valued function. The coefficients a, b, c, k and α are all real valued constants. It needs to be noted that Eqs. (3.1), 3.2 was already studied by the semi-inverse variational principle and ansatz method earlier in 2010 and 2011 respectively [30,31].

In order to solve (3.1) and (3.2) the following traveling wave hypothesis is picked:

$$q(x,t) = g(x - vt)e^{i\phi} \tag{3.3}$$

and

$$r(x,t) = h(x - vt) \tag{3.4}$$

where the phase component $\phi(x,t)$ is defined as

$$\phi(x,t) = -\kappa x + \omega t + \theta \tag{3.5}$$

In (3.3), (3.4), (3.5), v represents the velocity of the wave, while g and h are the wave profiles. From the phase component, κ is the frequency, ω is the wave number and θ is the phase constant. Now, substituting (3.3), (3.4), (3.5) into (3.1) and (3.2) and decomposing into real and imaginary parts give

$$v = -2a\kappa \tag{3.6}$$

$$ag'' - (\omega + a\kappa^2)g + bg^{2n+1} = cgh \tag{3.7}$$

and

$$h = \frac{\alpha}{v^2 - k^2}g^{2n}. \tag{3.8}$$

where $g' = dg/ds$ and $g'' = d^2g/ds^2$ and

$$s = x - vt. \tag{3.9}$$

Here, the velocity of the wave given by (3.6) is obtained from the imaginary part while (3.7) is obtained from the real part. Now, from (3.7) and (3.8),