



Modification of Block Pulse Functions and their application to solve numerically Volterra integral equation of the first kind

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ABSTRACT

In this paper a modification of Block Pulse Functions is introduced and used to solve Volterra integral equation of the first kind. Some theorems are included to show convergence and advantage of the method. Some examples show accuracy of the method.

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1. Introduction

Approximation theory is concerned with how functions can best be approximated with simpler functions called base functions and with quantitatively characterizing the errors introduced thereby [1]. One of these base functions is Block Pulse Functions (BPFs) [2] on which some researches are based. However BPFs are very common in use, it seems their convergence is weak and some published papers have tried to improve the speed of BPFs convergence with different methods like hybrid BPFs [3–5]. In fact by referring to error bound of BPFs approximation it seems for achieving double precision, number of BPFs have to be doubled which means solving systems of equations with double unknowns and double equations [1,6].

In this paper ε Modified Block Pulse Functions (ε MBPFs) are introduced and some theorems prove if ε MBPFs be used for achieving numerical expansions with k times more precision, there is no need to increase the number of BPFs, k times, which leads to solve a system of equations with k times more equations and unknowns. But the results of BPFs solution can be combined with solutions of $k - 1$ systems of equations with one more unknown and nearly achieve k times more precision.

We use ε MBPFs and directly solve Volterra integral equation of the first kind, then by some examples we show the efficiency of ε MBPFs.

2. Block Pulse Functions (BPFs)

BPFs are studied by many authors and applied for solving different problems, for example see [2–7].

Definition: An m -set of BPFs is defined over the interval $[0, T)$ as

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$$\psi_i(t) = \begin{cases} 1, & \frac{iT}{m} \leq t < \frac{(i+1)T}{m} \\ 0, & \text{otherwise,} \end{cases} \quad (2.1)$$

where $i = 0, \dots, m-1$ with m as a positive integer. Also, $h = T/m$, and ψ_i is the i th BPF.

In this paper it is assumed that $T = 1$, so BPFs are defined over $[0, 1)$ and $h = 1/m$.

There are some properties for BPFs, the most important properties are disjointness, orthogonality, and completeness.

The disjointness property can be clearly obtained from the definition of BPFs:

$$\psi_i(t)\psi_j(t) = \begin{cases} \psi_i(t), & i = j, \\ 0, & i \neq j, \end{cases} \quad (2.2)$$

where $i, j = 0, \dots, m-1$.

The other property is orthogonality. It is clear that

$$\int_0^1 \psi_i(t)\psi_j(t)dt = h\delta_{ij}, \quad (2.3)$$

where δ_{ij} is Kronecker delta.

The third property is completeness. For every $f \in L^2([0, 1))$ when m approaches to infinity, Parseval's identity holds:

$$\int_0^1 f^2(t)dt = \sum_{i=0}^{\infty} f_i^2 \|\psi_i(t)\|^2, \quad (2.4)$$

where

$$f_i = \frac{1}{h} \int_0^1 f(t)\psi_i(t)dt. \quad (2.5)$$

Vector forms: Consider the first m terms of BPFs and write them concisely as m -vector:

$$\Psi(t) = [\psi_0(t), \dots, \psi_{m-1}(t)]^T, \quad t \in [0, 1).$$

The above representation and disjointness property follows [2]:

$$\Psi(t)\Psi^T(t) = \begin{bmatrix} \psi_0(t) & 0 & \cdots & 0 \\ 0 & \psi_1(t) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \psi_{m-1}(t) \end{bmatrix}, \quad (2.6)$$

$$\Psi^T(t)\Psi(t) = 1, \quad (2.7)$$

$$\Psi(t)\Psi^T(t)V = \tilde{V}\Psi(t), \quad (2.8)$$

where V is an m -vector and $\tilde{V} = \text{diag}(V)$. Moreover, it can be clearly concluded that for an $m \times m$ matrix B :

$$\Psi^T(t)B\Psi(t) = \hat{B}^T\Psi(t), \quad (2.9)$$

where $\hat{B}$ is an m -vector with elements equal to the diagonal entries of matrix B .

BPFs expansion: The expansion of a function $f(t)$ over $[0, 1)$ with respect to $\psi_i(t)$, $i = 0, \dots, m-1$ may be compactly written as:

$$f(t) \simeq \sum_{i=0}^{m-1} f_i \psi_i(t) = F^T \Psi(t) = \Psi^T(t)F, \quad (2.10)$$

where $F = [f_0, \dots, f_{m-1}]^T$ and f_i 's is defined by (2.5).

Now assume $k(s, t) \in L^2([0, 1) \times [0, 1))$. It can be expanded with respect to BPFs as

$$k(s, t) \simeq \Psi^T(s)K\Gamma(t), \quad (2.11)$$

where $\Psi(s)$ and $\Gamma(t)$ are m_1 and m_2 components BPFs vectors, respectively, and K is the $m_1 \times m_2$ block pulse coefficient matrix with k_{ij} , $i = 0, \dots, m_1 - 1$, $j = 0, \dots, m_2 - 1$, as:

$$k_{ij} = m_1 m_2 \int_0^1 \int_0^1 k(s, t) \psi_i(s) \gamma_j(t) ds dt. \quad (2.12)$$

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