Commun Nonlinear Sci Numer Simulat 19 (2014) 4003–4012

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Ghost-vibrational resonance

S. Rajamani [a], S. Rajasekar [a,*], M.A.F. Sanjuán [b]

[a] School of Physics, Bharathidasan University, Tiruchirappalli 620 024, India
[b] Nonlinear Dynamics, Chaos and Complex Systems Group, Departamento de Física, Universidad Rey Juan Carlos, Tulipán s/n, 28933 Móstoles, Madrid, Spain

ARTICLE INFO

Article history:
Received 26 March 2014
Accepted 11 April 2014
Available online 24 April 2014

Keywords:
Ghost resonance
Multi-frequency signal
Duffing oscillator

ABSTRACT

Ghost-stochastic resonance is a noise-induced resonance at a fundamental frequency missing in the input signal. We investigate the effect of a high-frequency, instead of a noise, in a single Duffing oscillator driven by a multi-frequency signal $F(t) = \sum_{i=1}^{n} f_i \cos(\omega_i + \Delta\omega_0)t$, $\omega_i = (k + i - 1)\omega_0$, where k is an integer greater than or equal to two. We show the occurrence of a high-frequency induced resonance at the missing fundamental frequency ω_0. For the case of the two-frequency input signal, we obtain an analytical expression for the amplitude of the periodic component with the missing frequency. We present the influence of the number of forces n, the parameter k, the frequency ω_0 and the frequency shift $\Delta\omega_0$ on the response amplitude at the frequency ω_0. We also investigate the signal propagation in a network of unidirectionally coupled Duffing oscillators. Finally, we show the enhanced signal propagation in the coupled oscillators in absence of a high-frequency periodic force.

1. Introduction

Response of nonlinear systems to a harmonic force with a single frequency has been investigated in detail. A non-monotonic variation of the amplitude of the response occurs [1,2], in a typical nonlinear system when the frequency of the driving force is varied. In particular, the oscillation amplitude of the system output increases with the increase in the frequency of the external force, it reaches a maximum at a particular frequency and then it decreases with further increase in the frequency. This resonance phenomenon is widespread and has been utilized in several devices. In bistable and multistable systems when the amplitude of the external periodic force is below a threshold (that is, there is no switching motion between the coexisting stable states), then a transition between the coexisting states can be induced by a weak noise. At an appropriate optimum noise intensity, almost a periodic switching between coexisting states occurs resulting in a maximum system response. This noise-induced resonance phenomenon is termed as *stochastic resonance* [3,4]. Resonance can be realized when the noise term is replaced by a high-frequency periodic force and is called *vibrational resonance* [5,6]. Furthermore, it is possible to generate a chaotic signal that mimics the probability distribution of the Gaussian white noise. Such a signal can also give rise to a resonant effect analogous to the noise-induced resonance and is called *chaotic resonance* [7]. In all the above resonance phenomena, in absence of a resonance inducing source, the system is driven by a weak harmonic force with a single frequency. There are signals with multiple frequencies. Examples include human speech, musical tones and square-waves. Design of an approximate multi-frequency signal is very important in minimizing the nonlinear distortion in the multi-frequency system identification methods [8,9].

* Corresponding author. Tel.: +91 431 2407057; fax: +91 431 2407093.
E-mail addresses: rajeebard@gmail.com (S. Rajamani), rajasekar@cnld.bdu.ac.in (S. Rajasekar), miguel.sanjuan@urjc.es (M.A.F. Sanjuán).

http://dx.doi.org/10.1016/j.cnsns.2014.04.006
1007-5704/

Chialvo et al. [10,11] investigated the response of a threshold device to an input signal containing several frequencies in the presence of noise. When the frequencies of the driving force are of a higher-order of a certain fundamental frequency, then the system is found to show a maximum response at the missing fundamental frequency at an optimum noise intensity. This fundamental frequency, which is absent in the input signal, detected by the device is called ghost-frequency and the underlying resonance phenomenon is termed as *ghost-stochastic resonance* [10,11]. When the input signal is set into an anharmonic by introducing a same frequency shift to all the harmonic terms, the system is found to show a resonance at a certain shifted frequency. This ghost resonance phenomenon can be used to explain the missing fundamental illusion in which a third lower pitched tone is often heard when two tones occur together [11].

The occurrence of a ghost resonance induced by noise has been analysed mostly in excitable systems. For example, it was found in the sudden dropouts exhibited by a semiconductor laser [12], two laser systems coupled bidirectionally [13], vertical-cavity surface emitting lasers [14], monostable Schmitt trigger electronic circuit [15], an excitable Chua's circuit [16], a chaotic Chua's circuit [17] and a system of n-coupled neurons [18]. Subharmonic resonance behavior in a nonlinear system with a multi-frequency force containing the fundamental frequency in the absence of a high-frequency input signal is studied in [19].

Because nonlinear systems with double-well and multi-well potentials are wide-spread it is foremost important to investigate the response of these systems to the multi-frequency force and analyse the occurrence of ghost resonance in them and also with sources other than external noise. Motivated by the above considerations, in the present work, we explore the possibility of a ghost resonance induced by a high-frequency deterministic force rather than a noise. We consider the Duffing oscillator driven by multi-frequency force $F(t)$ and a high-frequency force $g\cos\Omega t$. The multi-frequency force $F(t)$ is given by

$$F(t) = \sum_{i=1}^{n} f_i \cos(\omega_i + \Delta\omega_0)t, \quad \omega_i = (k+i-1)\omega_0 \tag{1}$$

with $k \geqslant 2$ and $\Omega \gg \omega_n (= (k+n-1)\omega_0)$. We begin our analysis with $n=2,\ k=2$ and $\Delta\omega_0 = 0$. We show the occurrence of a resonance at the fundamental frequency ω_0 missing in the input signal $F(t)$. The value of g at which the resonance at the frequency ω_0 occurs, increases monotonically while the value of the response amplitude $Q(\omega_0)$ at resonance decreases with ω_0. Interestingly, the case of $n=2$ by applying a theoretical method, we are able to obtain an approximate analytical expression for the response amplitudes $Q(\omega_i),\ i=0,1,2$. Theoretical results are in good agreement with the numerical predictions. We study the influence of the number of periodic forces n, the parameters k and g and the frequency shift $\Delta\omega_0$ on $Q(\omega_0)$. For values of $k>2$ or $\Delta\omega_0 \neq 0$, the response amplitude $Q(\omega_0)$ becomes 0 when the oscillation center of the orbit is at the origin and this happens for g values above a certain critical value.

Next, we consider a network of unidirectionally coupled N-Duffing oscillators with the multi-frequency force and the high-frequency force applied to the first oscillator only. The first system is uncoupled. The coupling term is chosen to be linear. We denote $Q_i(\omega_0)$ as the response amplitude of the ith oscillator at the frequency ω_0. For a coupling strength above a critical value, an undamped signal propagation, that is, $Q_N(\omega_0) > Q_1(\omega_0)$ occurs at the missing fundamental frequency, even in the absence of the high-frequency periodic force. Interestingly, in the undamped signal propagation case, the response amplitude increases with the unit number i and then becoming a constant. The saturation value of Q is found to be independent of the parameters $k,\ n$ and $\Delta\omega_0$ in $F(t)$. Finally, we consider a network of unidirectionally coupled oscillators, where all the oscillators are driven by the external forces.

2. Resonance in a single Duffing oscillator

We consider the equation of motion of the Duffing oscillator driven by n harmonic forces $F(t)$ given by Eq. (1) and the high-frequency periodic force $g\cos\Omega t$ as

$$\ddot{x} + d\dot{x} + \alpha x + \beta x^3 = F(t) + g\cos\Omega t. \tag{2}$$

Throughout our study we fix the values of the parameters as $d=0.5,\ \alpha=-2,\ \omega_0=0.5,\ \beta=1,\ \Omega=30\omega_0$ and treat g as the control parameter. The potential associated to the system in the absence of damping and external force is of a double-well form, since $\alpha<0$ and $\beta>0$.

2.1. Numerical analysis

From the numerical solution of Eq. (2), we compute the sine and cosine components $Q_s(\omega)$ and $Q_c(\omega)$ respectively of the solution at various frequencies in the interval $\omega \in [0, 20]$ using the equations

$$Q_s(\omega) = \frac{2}{NT}\int_0^{NT} x(t)\sin\omega t\,dt, \tag{3a}$$

$$Q_c(\omega) = \frac{2}{NT}\int_0^{NT} x(t)\cos\omega t\,dt, \tag{3b}$$

where $T=2\pi/\omega$ and N is say 500. Then $Q(\omega) = \sqrt{Q_s^2+Q_c^2}/f$ with $f=(1/n)\sum_{i=1}^{n} f_i$.