Commun Nonlinear Sci Numer Simulat 16 (2011) 10–20

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Short communication

Global synchronization in arrays of coupled Lurie systems with both time-delay and hybrid coupling

Tao Li [a,*], Aiguo Song [a], Shumin Fei [b], Ting Wang [b]

[a] School of Instrument Science and Engineering, Southeast University, Nanjing 210096, China

[b] Key Laboratory of Measurement and Control of CSE (School of Automation, Southeast University), Ministry of Education, Nanjing 210096, China

ARTICLE INFO

Article history:
Received 7 November 2009
Received in revised form 22 January 2010
Accepted 12 April 2010
Available online 18 April 2010

Keywords:
Lurie systems
Delay-dependent synchronization criterion
Time-varying delay
Hybrid coupling
LMI approach

ABSTRACT

In this paper, we propose and study an array of coupled delayed Lurie systems with hybrid coupling, which is composed of constant coupling, state delay coupling, and distributed delay coupling. Together with Lyapunov–Krasovskii functional method and Kronecker product properties, two novel synchronization criteria are presented within linear matrix inequalities based on generalized convex combination, in which these conditions are heavily dependent on the upper and lower bounds of state delay and distributed one. Through adjusting inner coupling matrix parameters in the derived results, we can realize the designing and applications of the addressed systems by referring to Matlab LMI Toolbox. The efficiency and applicability of the proposed criteria can be demonstrated by three numerical examples with simulations.

1. Introduction

Presently, it is widely known that synchronization is an important property of dynamical systems, which can be observed in natural, social, physical and biological systems, and has found many applications in various engineering fields, such as secure communication [1], image processing [2], and harmonic oscillation generation [3]. Also, the existence of synchronization in language emergence and development results can help come up with the common vocabulary and agents' synchronization in organization management can improve their work efficiency. Thus in recent years, due to the great potential applications, the problem on synchronization in various dynamical systems has been extensively studied in the present literature, see [9–12,16–29] and references therein.

Many nonlinear control systems, such as Chua's circuits [4], Goodwin model [5] and Swarm model [6], can be categorized as Lurie systems, which are regarded as a feedback connection of linear dynamical system and nonlinear elements with the nonlinear ones satisfying some restrictions. Thus since Lurie and Postnikov first proposed the concept and analysis on absolute stability, researchers have attached much importance to dynamical analysis of Lurie systems, and many elegant results have been proposed [7–12,26]. Furthermore, it has been shown that as some conditions were satisfied, a chaotic system (the slave system) might become synchronized to another identical one (the master system) if the master system sent some driving signals to the slave one. On the other hand, time-delays are frequently encountered in various control engineering systems, which can make the control analysis and design much more complicated. Thus many researchers have paid much attention to master–slave synchronization for Lurie systems, especially for the delayed Lurie systems [9–12].

Meanwhile, since chaos synchronization in arrays of linearly coupled chaotic systems was first introduced by [13], arrays of coupled dynamical systems including the delayed ones have attracted many researchers' attention as they can exhibit

* Corresponding author.
E-mail addresses: fengtailitao@yahoo.com.cn (T. Li), a.g.song@seu.edu.cn (A. Song).

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doi:10.1016/j.cnsns.2010.04.008

some interesting phenomena [14,15], and a large number of elegant methods have been proposed in [16–29]. However, it has come to our attention that few researchers have considered the global synchronization for arrays of delayed Lurie systems with couplings. Though the authors have made some initial efforts on constant coupling for such problem, the derived results in [26] cannot tackle the variable delay, which means that their methods need great improvements. Meanwhile, considering the existence of delay in spreading due to the finite speed of transmission and traffic congestion, delay coupling should receive much attention, i.e., there exist coupling terms between the state variables and the delayed ones, in which the delay ones can describe the decentralized nature of real-word coupled Lurie systems. On the other hand, a Chua's system is designed with existence of distributed delays and it has been observed that a Lurie system has a spatial nature due to the presence of a number of parallel ways of axon sizes and lengths. Hence during discussing the synchronization of coupled Lurie systems, the constant coupling, delay coupling, and distributed delay coupling should be taken into consideration altogether, which led to the introduction of hybrid coupling in [29]. Therefore, it is urgent and challenging to establish some less conservative and easy-to-check criteria ensuring the global synchronization for arrays of coupled Lurie systems with time-varying delay, which constitutes the main focus of this presented work.

In this paper, we make great efforts to investigate the global synchronization for arrays of coupled Lurie systems with both variable delay and hybrid coupling, in which the hybrid coupling was first introduced in [29]. Through constructing an augmented Lyapunov–Krasovskii functional and using appropriate integral inequality, two novel sufficient conditions are presented in terms of LMIs, whose feasibility can be easily checked by resorting to Matlab LMI Toolbox. In particular, the upper bound on derivative of state delay can take any value. Finally, the effectiveness of the proposed criteria can be demonstrated by utilizing three numerical examples.

Notations. $\mathbf{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbf{R}^{n\times m}$ is the set of all $n\times m$ real matrices. For the symmetric matrices X, Y, $X>Y$ (respectively, $X\geqslant Y$) means that $X-Y>0$ ($X-Y\geqslant 0$) is a positive-definite (respectively, positive-semi-definite) matrix; A^T stands for the transpose of matrix A; I represents the identity matrix of an appropriate dimension; $A\otimes B$ indicates the Kronecker product of the $m\times n$ matrix A and $p\times q$ matrix B; and $\begin{bmatrix} X & Y \\ Y^T & Z\end{bmatrix}=\begin{bmatrix} X & Y \\ * & Z\end{bmatrix}$ with $*$ denoting the symmetric term in a symmetric matrix.

2. Problem formulations

Suppose that the nodes are coupled with states $x_i(t)$, $i\in\{1,\ldots,N\}$, then the Lurie systems with mixed time-varying delays can be described by

$$\begin{aligned}\dot{x}_i(t)&=Ax_i(t)+Bx_i(t-\tau(t))+Cf\Big(E^Tx_i(t)\Big)+Dh\Big(G^Tx_i(t-\tau(t))\Big)+\sum_{j=1,j\neq i}^{N}l_{ij}K_1\left[x_j(t)-x_i(t)\right]\\&\quad+\sum_{j=1,j\neq i}^{N}l_{ij}K_2\left[x_j(t-\tau(t))-x_i(t-\tau(t))\right]+\sum_{j=1,j\neq i}^{N}l_{ij}K_3\int_{t-v(t)}^{t}\left[x_j(s)-x_i(s)\right]ds,\end{aligned}\tag{1}$$

in which $x_i(t)=[x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t)]^T$ are the state vectors; here $A=[a_{ij}]_{n\times n}$, $B=[b_{ij}]_{n\times n}$, $C=[c_{ij}]_{n\times n_1}$, $D=[d_{ij}]_{n\times n_2}$, $E=[e_{ij}]_{n\times n_1}=[e_1,e_2,\ldots,e_{n_1}]$, $G=[g_{ij}]_{n\times n_2}=[g_1,g_2,\ldots,g_{n_2}]$; $K_1=[k_{1ij}]_{n\times n}$, $K_2=[k_{2ij}]_{n\times n}$, $K_3=[k_{3ij}]_{n\times n}$ are the inner coupling matrices between the connected nodes i and j at time t and $t-\tau(t)$, respectively; $f\Big(E^Tx_i(t)\Big)=\Big[f_1\left(e_1^Tx_i(t)\right),\ldots,f_{n_1}\left(e_{n_1}^Tx_i(t)\right)\Big]^T$, and $h\Big(G^Tx_i(t-\tau(t))\Big)=\Big[h_1\Big(g_1^Tx_i(t-\tau(t))\Big),\ldots,h_{n_2}\Big(g_{n_2}^Tx_i(t-\tau(t))\Big)\Big]^T$ denote the time-variant piecewise continuous nonlinearities satisfying $f(0)=0$ and $h(0)=0$.

For the dynamical system (1), the following assumptions are utilized throughout this paper.

A1. Here $\tau(t)$, $v(t)$ denote two interval time-varying delays satisfying

$$0\leqslant\tau_0\leqslant\tau(t)\leqslant\tau_m,\quad \dot{\tau}(t)\leqslant\mu<+\infty,\qquad 0\leqslant v_0\leqslant v(t)\leqslant v_m.\tag{2}$$

Here we set $\bar{\tau}_m=\tau_m-\tau_0$ and $\bar{v}_m=v_m-v_0$.

A2. $L=[l_{ij}]_{N\times N}$ is the configuration matrix that is irreducible and satisfies the following condition:

$$l_{ij}=l_{ji},\quad i\neq j,\qquad l_{ii}=-\sum_{j=1,j\neq i}^{N}l_{ij},\tag{3}$$

here $l_{ij}>0$, if there is a connection between node i and the one j and otherwise, $l_{ij}=0$.

A3. For all $\alpha,\beta\in\mathbf{R}$, the nonlinear functions $f_i(\cdot)$, $h_j(\cdot)$ satisfy $f_i(0)=0$, $h_j(0)=0$, and

$$\begin{aligned}&\left[f_i(\alpha)-f_i(\beta)-\sigma_i^+(\alpha-\beta)\right]\left[f_i(\alpha)-f_i(\beta)-\sigma_i^-(\alpha-\beta)\right]\leqslant 0,\qquad i=1,\ldots,n_1,\\&\Big[h_j(\alpha)-h_j(\beta)-\delta_j^+(\alpha-\beta)\Big]\Big[h_j(\alpha)-h_j(\beta)-\delta_j^-(\alpha-\beta)\Big]\leqslant 0,\qquad j=1,\ldots,n_2,\end{aligned}$$