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Model reduction on inertial manifolds for N–S equations approached by multilevel finite element method

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A B S T R A C T

Approximate Inertial Manifolds (AIMs) is approached by multilevel finite element method, which can be referred to as a Post-processed nonlinear Galerkin finite element method, and is applied to the model reduction for fluid dynamics, a typical kind of nonlinear continuous dynamic system from viewpoint of nonlinear dynamics. By this method, each unknown variable, namely, velocity and pressure, is divided into two components, that is the large eddy and small eddy components. The interaction between large eddy and small eddy components, which is negligible if standard Galerkin algorithm is used to approach the original governing equations, is considered essentially by AIMs, and consequently a coarse grid finite element space and a fine grid incremental finite element space are introduced to approach the two components. As an example, the flow field of incompressible flows around airfoil is simulated numerically and discussed, and velocity and pressure distributions of the flow field are obtained accurately. The results show that there exists less essential degrees-of-freedom which can dominate the dynamic behaviors of the discretized system in comparison with the traditional methods, and large computing time can be saved by this efficient method. In a sense, the small eddy component can be captured by AIMs with fewer grids, and an accurate result can also be obtained.

1. Introduction

Most of dynamic systems encountered in engineering are nonlinear continuous dynamic systems, which have a rich variety of nonlinear phenomena, such as the large and complex fluid–structure interaction systems, aerodynamics, thin-walled structures with large deformation, etc. Normally, the finite element method or other numerical methods are applied to the approximation of the governing equations, due to the difficulty of obtaining a solution in analytical form. As the results, the resulting equations are generally nonlinear dissipative evolution equations with many degrees-of-freedom in sense of dynamics. For such kind of equations, there are several classical numerical schemes to approximate them, such as Newmark, Wilson-θ, Houbolt and the Runge–Kutta scheme with higher precision if the system is transferred into phase space. However, great difficulties will arise from analyzing the nonlinear dynamics both qualitatively and quantitatively in a finite-dimensional phase space of higher dimension [1]. For example, the analysis of nonlinear dynamical systems, using the numerical schemes mentioned above, requires considerable computing time due to the large number of degrees-of-freedom, and some numerical round-off errors will have a strong influence on the long-term behaviors of the systems or the bifurcation analysis if the systems have a cluster of bifurcation points [2–5]. In other words, model reduction is the key to such obstacle and currently urgent for the bifurcation analysis by large-scale

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numerical computation. Therefore, it is natural to reduce the model from higher to lower dimensions and to achieve an acceptable approximation of the original dynamics before large-scale numerical analysis is applied to the original system. Indeed, this reduction technique can be reached for some certain dissipative systems, by neglecting inessential degrees-of-freedom of the system and keeping the topology of the solutions unchanged [1]. Under such background, a number of researchers have developed many practical numerical algorithms [2]. For the linear dynamic system, the component mode synthesis techniques can be used to analyze the dynamic behaviors of the system, and much computing time will be saved with an acceptable approximate result. However, for the nonlinear dynamic system, there are a few methods for the model reduction. Most of the numerical algorithms are developed based on the component mode synthesis techniques which can be used for linear dynamical systems with acceptable approximate results, while few rigorous theoretical studies or the error estimate has been carried out on the influence of such reduction on the long-term behaviors, though a lot of numerical experiments are given [6–10]. Strictly speaking, due to the strong nonlinearities of some dissipative autonomous dynamical systems, the reduction of the systems has a greater influence on the solution at a certain degree, in a mathematically precise way [11,12].

Fluid dynamics, a typical kind of continuous dynamic system, is governed by a set of nonlinear dissipative evolutionary equations, and there are many nonlinear phenomena, such as separation in boundary layer, soliton and turbulence, and some other open problems in it. In particular, the connections between fluid mechanics, partial differential equations and nonlinear dynamical system, and the global attractors and turbulence, are the essential heart of understanding of many important problems of natural science. There is a number of numerical analysis of Navier–Stokes equations based on Finite Element Method, and almost of them are the adaptations of traditional *Galerkin* procedure [1]. However, an important deficiency of the existing numerical methods in the computation fluid dynamics is the cost of computing time, that is, there are a large number of degrees-of-freedom after the system is approached by the discretization, and the system is the one with higher dimension from viewpoint of nonlinear dynamics. Hence, in the nonlinear continuous dynamic systems, it is the current aim to reduce the original system to a system with less degrees-of-freedom.

On the other hand, it is well known that the asymptotic behaviors of some higher dimensional or even infinite-dimensional dissipative dynamic systems evolve to a compact set known as a global attractor, which is finite-dimensional [13]. That means such kind of dynamic systems can be described by the deterministic flow on a lower dimensional attractor. Hence, it opens the way for the reduction of the dynamics of infinite-dimensional dissipative equations to finite-dimensional systems, namely, higher dimensional dissipative equations to a lower dimensional system. Consequently, various schemes have been used to construct a finite system for reproducing the asymptotic dynamics of the original dynamic system [14–19]. One of the schemes is the traditional *Galerkin* method, which ignores the small spatial structure of a solution. However, an important and well-known aspect of nonlinear dynamics is the sensitive dependence of the solution on the perturbations. Such perturbations include small variations in initial and boundary values, as well as numerical errors if a numerical computation method is adopted. A slight perturbation to the system may produce very important effects and significant changes in the system's configuration after a long time [20]. The Center Manifold Theory can be applied to the system with a small number of modes, whose eigenvalues are close to the imaginary axis, but a small parameter variation from the critical value or a large parameter variation for some cases will have the effect that additional modes will become unstable, and the originally low dimensional system will not be valid anymore [1].

The theory of Inertial Manifolds (IMs) has shown that the long time dynamic behaviors of dissipative partial differential equation (PDE) can be fully described by that of a set finite ordinary differential equation to which the PDE is reduced on IMs. Roughly speaking, the IMs can be considered as a reduction method. In fact, the methods used to construct the IMs are the adaptations of various theories in the studies of center manifolds and integral manifolds. Then, the stability and bifurcation can be investigated based on the ordinary differential equation with relatively low dimension on the IMs, and some natures of nonlinear phenomena can be explained availably and feasibly.

For decades, the theory of Inertial Manifolds is a technique for model reduction [1,21,22]. Unfortunately, the existence of IMs usually holds only under the very restrictive spectral gap condition. Hence in practical applications the concept of Approximate Inertial Manifolds (AIMs) has been introduced [12,22,23]. However, there are few studies on the IMs and AIMs for the second order in time nonlinear dissipative autonomous dynamic systems. In [24], AIMs for second order in time partial differential equations with delay are constructed, and some important results have been given. In light of AIMs developed in infinite dynamic systems and Mode Analysis in linear structural dynamics, a combined method is presented to reduce the second order in time nonlinear dissipative autonomous dynamic systems with many degrees-of-freedom, which are encountered frequently in engineering, and the influence of model reduction on the long-term behaviors has been studied in detail, and the error estimate is also given [25]. Additionally, the AIMs has been introduced to the dynamic snap-through buckling analysis of shallow arches under impact load [26].

In this study, the multilevel finite element method is developed to approach the AIMs for Navier–Stokes equations, and the governing equations can be projected onto the space dominated by the Inertial Manifolds, which is finite dimension.

2. Governing equations and multilevel finite element method

For the sake of simplicity, the AIMs approached by multilevel finite element method is given for the incompressible flow. The governing equations with elementary variables for the incompressible flow are