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Numerical computation of pulsatile flow through a locally constricted channel

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ABSTRACT

This paper deals with the numerical solution of a pulsatile laminar flow through a locally constricted channel. A finite difference technique has been employed to solve the governing equations. The effects of the flow parameters such as Reynolds number, flow pulsation in terms of Strouhal number, constriction height and length on the flow behaviour have been studied. It is found that the peak value of the wall shear stress has significantly changed with the variation of Reynolds numbers and constriction heights. It is also noted that the Strouhal number and constriction length have little effect on the peak value of the wall shear stress. The flow computation reveals that the peak value of the wall shear stress at maximum flow rate time in pulsatile flow situation is much larger than that due to steady flow. The constriction and the flow pulsation produce flow disturbances at the vicinity of the constriction of the channel in the downstream direction.

1. Introduction

Investigation of oscillatory motion and the corresponding flow dynamics including flow separation phenomenon have important practical applications in many branches of engineering, in environmental related flow problems and in human cardiovascular system. Atherosclerosis is the most common form of arteriosclerosis i.e., hardening and loss of elasticity of the arterial wall specially due to deposition of plaque. The disease is characterized by thickening of the intima with plaques that can contain lipid-laden macrophages ("foam cells"). Several studies suggested that the development of arteriosclerosis is known to be closely related with presence of a locally irregular flow, variation of wall shear stress and the consequence of boundary layer separation. The vorticity layer formation that causes an unnatural distribution of wall shear stress [1] is the prime factor for the initiation and progression of the arterial diseases. Separation of flow can also contribute to pressure loss and is a major factor for lower percent stenoses. For a moderate level of stenosis, the critical Reynolds number is only 10 and thus separation will occur in most arterial stenoses [2].

Bifurcation theory in viscous fluid mechanics is a common phenomena and one of the simplest examples is the Coanda effect. In a channel with constriction or expansion, separation can occur in the lee of the constriction or expansion when the flow rate exceeds a critical value. If flow occurs through a symmetric expansion then separated region can form either side of the main flow. What is observed is that the two symmetric separated regions may not remain the samesize and the flow appears to attach to one or the other wall [3].

Several theoretical and experimental studies of fluid dynamics through differently geometries (constriction or expansion) have been carried out to evaluate the flow pattern and the shear stresses at the walls under steady and oscillatory flow

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conditions. The non-linear separated vorticity modifies the boundary layer structure and its separation region which eventually changes the whole flow dynamics. The dynamics of this kind of steady and pulsatile flow phenomena and corresponding flow separation has been extensively studied by several researchers e.g., Mahapatra et al. [4], Layek and Midya [5], Siouuffi [6], Pedrizzetti [7], Liu and Yamaguchi [8] etc. The technique of large-eddy simulation (LES) has been applied to the study of pulsatile flow through a modelled arterial stenosis by Mittal et al. [1], Tutty [9], Tutty and Pedley [10] etc.

In recent times, numerical solution of the Navier–Stokes equations in two-dimensional channel is straightforward, in particular for modest flow Reynolds numbers. The most popular numerical methods for solving incompressible Navier–Stokes equations are the primitive-variable formulation (i.e., based on velocity–pressure only) and streamfunction–vorticity technique. The staggered grid concept proposed by Harlow and Welch [11] has been employed in primitive-variable formulation due to avoid the decouple tendency of pressure in incompressible equations. The main difficulty in streamfunction–vorticity formulation is to find proper wall vorticity conditions and restricted to two-dimensional flows only.

In the present analysis we consider a simple two-dimensional planar geometrical model of a Newtonian viscous fluid through a constricted channel and the streamfunction–vorticity approach has been adopted. This simple model can include the important geometrical features of stenosed arteries and exhibits the many aspects of flow behaviour in the downstream of the stenosis. Blood flow in arteries is mostly dominated by unsteady flow phenomena and can be modelled approximately as a Newtonian fluid in particular to large vessels [12,13]. Since the development of atherosclerosis leads to the hardening of the arterial wall, the rigidity of the wall may be reasonably assumed. The fluid dynamics regarding post-stenotic flow may plays an important role in the initiation and progression of atherosclerosis and is therefore worthy of studies in the diagnosis of arterial disease. The main objective of the present work is to understand the flow behaviour in pulsatile flows in a constricted channel and investigate the effects of flow pulsation (in terms of Strouhal number st), constriction length ($2x_0$) and height (h) and Reynolds number (Re) on the flow quantities such as velocity distribution, wall shear stress and pattern of streamlines which are of practical importance.

2. Mathematical formulation of the problem

We consider here the pulsatile flow of an incompressible viscous fluid, with constant density ρ and kinematic viscosity ν through a channel having a locally symmetric constriction on both walls at the same location. The geometry of the constricted channel is shown in Fig. 1. Let (x^*, z^*) be the Cartesian coordinate of a material point in the system such that the x^*-axis is along the lower wall and z^*-axis is perpendicular to it, the origin O being taken at the middle of the constriction length ($2x_0$). The width of the channel is L everywhere except in the region of symmetric constriction on the lower ($z^* = g_1(x^*)$) and upper ($z^* = g_2(x^*)$) walls. The channel walls are being modelled mathematically as:

$$\begin{aligned} g_1(x^*) &= \frac{h^*}{2}\left[1 + cos(\pi x^*/x_0^*)\right], \quad |x^*| \leqslant x_0^* \\ &= 0, \quad |x^*| > x_0^* \\ g_2(x^*) &= L - \frac{h^*}{2}\left[1 + cos(\pi x^*/x_0^*)\right], \quad |x^*| \leqslant x_0^* \\ &= L, \quad |x^*| > x_0^* \end{aligned} \tag{1}$$

where h^* is the height of the constriction.

Introducing the following dimensionless quantities as

$$x = \frac{x^*}{L}, \quad z = \frac{z^*}{L}, \quad u = \frac{u^*}{U}, v = \frac{v^*}{U}, \quad t = \frac{t^*}{T}, \quad p = \frac{p^*}{\rho U^2}, \quad Re = \frac{UL}{\nu}, \quad st = \frac{L}{UT} \tag{2}$$

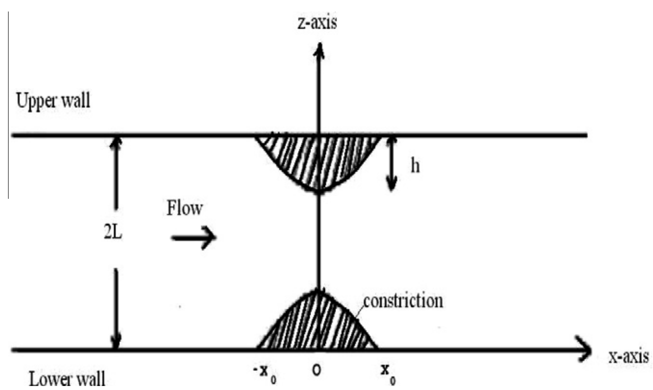

Fig. 1. Schematic diagram of the flow.