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Chaotic dynamics in classical nuclear billiards

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ABSTRACT

We consider several noninteracting nucleons moving in a 2D Woods–Saxon type potential well and hitting the vibrating surface. The Hamiltonian has a coupling term between the particle motion and the collective coordinate which generates a self-consistent dynamics. The numerical simulation is based on the solutions of the Hamilton equations which was solved using an algorithm of Runge–Kutta type (order 4–5) having an optimized step size, taking into account that the absolute error for each variable is less than 10^{-6}. Total energy is conserved with high accuracy, i.e., approx. 10^{-6} in absolute value. We analyze the chaotic behavior of the nonlinear dynamics system using phase-space maps, autocorrelation functions, power spectra, Lyapunov exponents and Kolmogorov–Sinai entropies. A qualitative and quantitative picture of the achievement of soft chaos is shown for a comparative study between the adiabatic and the resonance stage of nuclear interaction. We consider that the onset of chaos would be linked to the resonance stage of interaction. This assumption is argued in [1].

1. Introduction

Over the last two decades an increasing number of papers have treated the study of the deterministic chaotic behavior of Fermi nuclear systems. The order to chaos transition in the dynamics of independent classical particles in a container was first studied using computer simulations by Blocki et al. [2]. They analyzed the behavior of a gas of classical noninteracting particles enclosed on a multipole-deformed container which undergoes periodic shape oscillations and showed that higher multipolarities leads to chaotic motion. The destruction of order is paralleled by a transition from rubber-like to honey-like behavior of the independent particle nuclear model. Another step in this direction was done by Bauer et al. [3]. They have performed self-consistent calculations in semi-classical approximation utilizing a multipole–multipole interaction of the Bohr–Mottelson type for quadrupole and octupole deformations. In both cases the dynamical evolution showed a regular undamped collective motion which coexist with a weakly chaotic single-particle dynamics. Then Burgio et al. [4,5] considered a number of nucleons without spin and charge and with no internal structure which are moving in a 2D deep Woods–Saxon potential well and hitting the oscillating surface with a certain frequency. They discuss the dissipative behavior of the wall motion and its relation with the order to chaos transition in the dynamics of the microscopic degree of freedom. We developed those studies in [6] using informational Shannon entropy in order to study the nuclear fragmentation in the presence of dissipation. The parameters of the Woods–Saxon potential for different multipole deformations and their time dependence were taken into account in both, adiabatic and resonant regimes, in order to obtain information on nuclear fragmentation throughout a chaotic process. Thus, our numerical simulations have shown that every new interaction that allows

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supplementary couplings between particles decreases the onset of the nuclear fragmentation toward realistic nuclear interaction time scale.

We think that the interplay between chaos in single particle degree of freedom and regularity in collective coordinates may play a role in the time evolution of other physical systems as macroscopic fluid motion, plasmas in a tokamak and the human brain wave activity.

In Section 2, we present the model called "nuclear billiard". Section 3 is dedicated to the results of our simulations concerning the qualitative chaos analyses (autocorrelation, power spectra, and phase-space maps). In Section 4, we present the results of our quantitative chaos analyses. Section 5 presents the conclusions.

2. The toy model

We chose the system under study as an ensemble with N nucleons moving in a 2D Woods–Saxon potential well and hitting periodically the oscillating surface with a certain frequency. The ensemble of nucleons is considered as a Hamiltonian system therefore the total energy is conserved and this means that the walls can give energy to nucleons. The Hamiltonian of this kind of interaction in polar coordinates is [4,5]:

$$H(r_i,\theta_i,\alpha) = \sum_{i=1}^{N}\left(\frac{p_{r_i}^2}{2m} + \frac{p_{\theta_i}^2}{2mr_i^2} + V(r_i,R(\theta_i))\right) + \frac{p_\alpha^2}{2M} + \frac{1}{2}M\Omega^2\alpha^2 \tag{1}$$

where $\{p_{r_i}, p_{\theta_i}, p_\alpha\}$ are the conjugate momenta of the i(th) particle and of the collective coordinates $\{r_i,\theta_i,\alpha\}$, m = 938.5 MeV is the nucleon mass, Ω is the oscillating frequency of the collective variable α and $M = mNR_o^2$ is the Inglis mass. The Woods–Saxon potential is given by the function:

$$V(r_i,R(\theta_i)) = \frac{V_o}{1+e^{\frac{r_i-R(\theta_i)}{a}}} \tag{2}$$

with $V_o = -1500$ MeV and $a = 0.5$ fm. The potential is constant inside the billiard and rise very quickly on the surface. The nucleons were initially random distributed in the billiard having R_0 radius and the corresponding momenta were generated according to Maxwell distribution with a temperature $T = 36$ MeV. This value of temperature was chosen in order to mimic the Fermi motion of the particles [6].

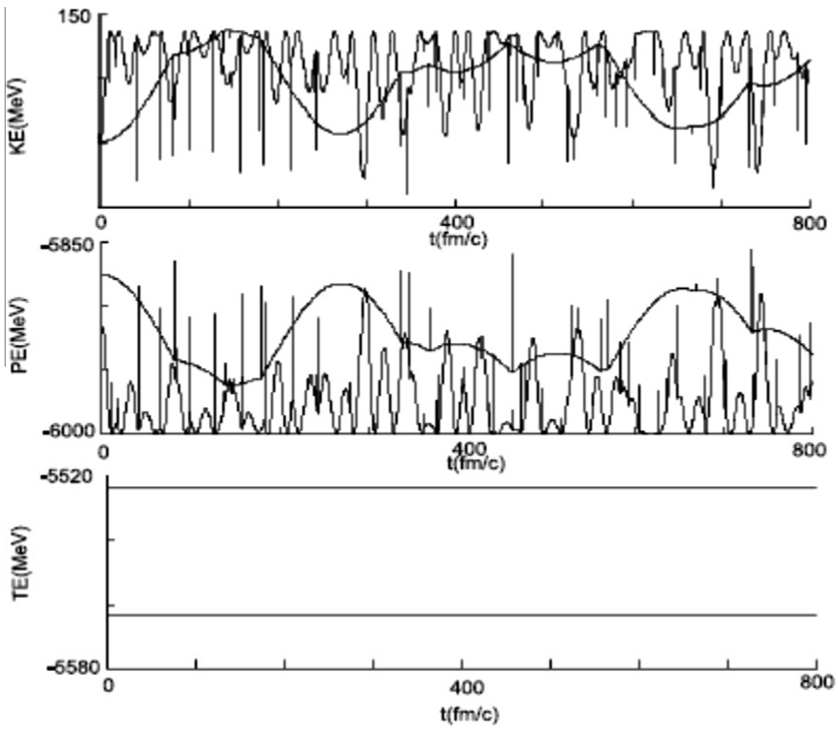

Fig. 1. Kinetic energy (KE), potential energy (PE) and total energy (TE) versus time for adiabatic ($\Omega_{ad} = 0.050$ c/fm) and resonant ($\Omega_{res} = 0.145$ c/fm) cases (N = 4, L = 0).