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LMI conditions for stability of impulsive stochastic Cohen–Grossberg neural networks with mixed delays ☆

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ABSTRACT

In this paper, the global asymptotic stability of impulsive stochastic Cohen–Grossberg neural networks with mixed delays is investigated by using Lyapunov–Krasovskii functional method and the linear matrix inequality (LMI) technique. The mixed time delays comprise both the multiple time-varying and continuously distributed delays. Some new sufficient conditions are obtained to guarantee the global asymptotic stability of the addressed model in the stochastic sense using the powerful MATLAB LMI toolbox. The results extend and improve the earlier publications. Two numerical examples are given to illustrate the effectiveness of our results.

1. Introduction

In 1983, Cohen–Grossberg [1] first introduced Cohen–Grossberg neural networks (CGNNs). It is well known that CGNNs have many important applications in various fields such as parallel computation, biological systems, information science and optimization problems, see [2,3,7–9], and the references therein. This model includes a lot of famous neural networks such as Hopfield neural network, Lotka–Volterra system, cellular neural network, recurrent neural network, see for example [4–6]. During the past several years, the dynamical behaviors of CGNNs have been extensively investigated by numerous scholars, and a large number of results have appeared in the literature, see [7–11,19–23,27,28].

In practice, a real system is usually affected by external perturbations which in many cases are of great uncertainty. Hence, it is necessary to consider the stochastic effects to the stability property of the neural networks, see [12–18]. Recently, some results have been proposed to guarantee the global asymptotic stability or exponential stability for stochastic CGNNs with constant or time-varying delays, see [21–23]. For example, in [20], Zhao and Ding investigated the almost sure exponential stability of stochastic CGNNs with constant delays by constructing suitable Lyapunov functional and employing the semimartingale convergence theorem. However, the results have only focused on constant delays. In [21], Su and Chen further obtained some results for robust asymptotic stability of stochastic CGNNs with discrete and distributed time-varying delays by using Lyapunov stability theory and LMI technology. Those results seem to be independent of the amplification functions. However, in order for $E\,\mathscr{L}V(x_t,t)$ to be negative for all initial data and $t \geqslant 0$, the effects of stochastic are ignored

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partly. In fact, when the time delays satisfy a weaker condition, the stability of stochastic CGNNs are mainly dependent of the random term, and therefore, stochastic effects are important and should be taken into account comprehensively.

On the other hand, as we known, artificial neural networks often are subject to impulsive perturbations which can affect dynamical behaviors of the systems, see [22–27]. Moreover, those perturbations often may make stable systems unstable or unstable systems stable. Therefore, impulsive effects should also be taken into account. Very recently, several initial results for stability of stochastic CGNNs with impulses have been proposed and studied [28,29]. In particular, Song and Wang [28] gave some criteria for the existence, uniqueness, and exponential p-stability of the equilibrium point for impulsive stochastic CGNNs with mixed delays by employing a combination of the M-matrix theory and stochastic analysis technique. The results improve and extend some of existing results [19,20,22]. However, it is worth noting that some of the conditions on delays and impulses are too restrictive and conservative and there still exists open room for further improvement.

Activated by the above discussions, the main aim of this paper is to investigate the global asymptotic stability of impulsive stochastic Cohen–Grossberg neural networks with mixed delays. By using Lyapunov–Krasovskii functional method and LMI technique, some results are obtained in terms of LMI, which can be easily calculated by MATLAB LMI toolbox. Our results extend and improve some of existing results [15,16,20,21]. Moreover, our results can be applied to the cases not covered in [28,29].

The rest of the paper is organized as follows. In Section 2, we introduce the impulsive stochastic Cohen–Grossberg neural networks models with mixed delays. The results for global asymptotic stability are stated in Section 3. In Section 4, we investigate the global robust stability of the addressed model. Some examples with simulations are discussed to illustrate the validity of the obtained results in Section 5. Finally, in Section 6, conclusions are drawn.

2. Problem description and preliminaries

In this paper, we will use the notation $\mathscr{A} > 0$ or $\mathscr{A} < 0$ to denote that the matrix $\mathscr{A}$ is a symmetric and positive definite or negative definite matrix. The notation $\mathscr{A}^T$ and $\mathscr{A}^{-1}$ mean the transpose of $\mathscr{A}$ and the inverse of a square matrix. If $\mathscr{A},\mathscr{B}$ are symmetric matrices, $\mathscr{A} > \mathscr{B}$ means that $\mathscr{A} - \mathscr{B}$ is positive definite matrix. I denotes the identity matrix. Moreover, the notation $\star$ always denotes the symmetric block in one symmetric matrix.

Consider the following stochastic Cohen–Grossberg neural networks model with multiple discrete time-varying and continuously distributed delays:

$$\begin{aligned} dx(t) = \Bigg\{ -\alpha(x(t))\Bigg[\beta(x(t)) - W^{(0)}f(x(t)) - \sum_{k=1}^{r} W^{(k)}f(x(t-\tau_k(t))) - B\int_{t-\mu(t)}^{t} f(x(s))ds - C\int_{-\infty}^{t} K(t-s)f(x(s))ds\Bigg]\Bigg\}dt \\ + \sigma(t, x(t), x(t-\tau_1(t)), \ldots, x(t-\tau_r(t)))d\omega(t) \end{aligned} \tag{1a}$$

and the corresponding model with impulses

$$\begin{cases} dx(t) = \Big\{ -\alpha(x(t))\Big[\beta(x(t)) - W^{(0)}f(x(t)) - \sum\limits_{k=1}^{r} W^{(k)}f(x(t-\tau_k(t))) - B\int_{t-\mu(t)}^{t} f(x(s))ds \\ \quad -C\int_{-\infty}^{t} K(t-s)f(x(s))ds\Big]\Big\}dt + \sigma(t, x(t), x(t-\tau_1(t)), \ldots, x(t-\tau_r(t)))d\omega(t), \quad t \neq t_k, \\ \Delta x(t_k) = J_k(x(t_k^-)), \quad k \in \mathbb{Z}_+, \end{cases} \tag{1b}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is the neuron state vector, $n \geqslant 2$ is the number of units in a neural network, $\alpha(x) = \mathrm{diag}(\alpha_1(x_1), \ldots, \alpha_n(x_n))$ represents an amplification function, $\beta(x) = (\beta_1(x_1), \ldots, \beta_n(x_n))^T$ is the behaved function, $B = (b_{ij})_{n\times n},\ C = (c_{ij})_{n\times n}$ and $W^{(k)} = \left(w_{ij}^{(k)}\right)_{n\times n},\ k = 0, 1, \ldots, r$, are the interconnection matrices, $f(\cdot)$ denotes the neuron activation, $\tau_k(t), \mu(t)$ represent the transmission delay with $0 \leqslant \tau_k(t) \leqslant \tau,\ \dot{\tau}_k(t) \leqslant \rho_k < 1$ and $0 \leqslant \mu(t) \leqslant \mu,\ \tau,\ \rho_k$ and μ are some real constants. $\omega(t) = (\omega_1(t), \ldots, \omega_n(t))$ is n-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathscr{F}, P)$ with a natural filtration $\{\mathscr{F}_t\}_{t\geqslant 0}$ generated by $\{\omega(s) : 0 \leqslant s \leqslant t\}$, where we associate Ω with the canonical space generated by $\omega(t)$, and denote by $\mathscr{F}$ the associated σ-algebra generated by $\omega(t)$ with the probability measure P. Moreover, $K(t) = \mathrm{diag}(k_1(t), \ldots, k_n(t))$ is the delay kernel function, k_i is a real value non-negative continuous function defined in $[0, \infty)$ satisfying

$$\int_0^{\infty} k_j(s)ds \doteq \kappa_j < \infty, \quad j = 1, 2, \ldots, n.$$

The initial conditions for system (1) are $x(t) = \varphi(t),\ t \leqslant 0,\ \varphi \in C^2_{\mathscr{F}_0}[(-\infty, 0), \mathbb{R}^n]$, where $C^2_{\mathscr{F}_0}$ denotes the family of all bounded $\mathscr{F}_0$-measurable, $C[(-\infty, 0), \mathbb{R}^n]$-valued random variables, satisfying $\|\varphi\| = \sup_{\theta\leqslant 0} E|\varphi(\theta)|^2 < \infty$, where E denotes the expectation of a stochastic process.

Let $C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}_+)$ denotes the family of all nonnegative functions $V(x, t)$ on $\mathbb{R}^n \times \mathbb{R}_+$ which are continuous once differentiable in t and twice differentiable in x. For each such V, we define an operator $\mathscr{L}V$ associated with (1) as