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Effect of vertical stratification on characteristics and energy of nonlinear internal solitary waves from a numerical model

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ABSTRACT

A numerical model is set up to study the impact of changes in vertical stratification on the properties of internal solitary waves (ISWs) generated by tidal flow over a ridge. Based on modifications of the observed stratification with a secondary thermocline over a main one in the South China Sea, the effects of five kinds of stratification on the characteristics and energy conversion of ISWs are investigated. In general, the isopycnal undergoing maximum displacement in ISWs is from slightly below the main thermocline. When the stratification below the ridge crest is reduced, the wave amplitude and the number of ISWs in a wave packet increase, while the phase speed, the wave half-width, the sum of ISW kinetic energy (KE) and available potential energy (APE) and the ratio of KE to APE decrease. When the stratification in the upper layer is reduced, the ISW amplitude, the number of ISWs, the phase speed and the sum of KE and APE decrease, while the wave half-width and the ratio of KE to APE increase. If the main thermocline is over the secondary one, the ISW amplitude, the wave half-width, the sum of KE and APE and the ratio of KE to APE increase, while the phase speed reduces. For stratification with two thermoclines, the ISW phase speed increases but the half-width decreases. In addition, the ratio of baroclinic to barotropic energy is found to be between 10% and 40%, and the ratio of ISW KE to APE is between 1.30 and 1.65. It is also shown that the ratio of KE to APE for the stratification with two thermoclines is about 2–6% larger than that for the stratification with only one thermocline. If the thermocline is lowered by about 30 m (0.064 of the total water depth), the ratio of KE to APE reduces by about 10%.

1. Introduction

In recent decades, oceanic observations of internal solitary waves (ISWs) have often been reported [1–3]. ISWs are usually generated by tidal flow over underwater bathymetry in the stratified ocean. There are several non-dimensional parameters governing the generation of ISWs by tidal flow over underwater ridge. Of the non-dimensional parameters governing the generation of ISWs, most attention has been paid to the relative steepness of topography [4,5],

$$\varepsilon = h_x \Big/ \sqrt{(\sigma^2 - f^2)/(N(z)^2 - \sigma^2)}, \tag{1}$$

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where $h_x = dh/dx$ is the topographic slope, σ is the tidal frequency, f is the Coriolis parameter and $N(z)$ is the buoyancy frequency. According to formula (1), it can be seen that both the structure of stratification and topography are important for governing the generation of ISWs by affecting the relative steepness ε. However, most of the previous work mainly pays attention to the effect of topographic characteristics on ISWs, while the study on the effect of stratification is relatively unexplored.

Actually, a study of the effects of changes in stratification on ISWs is also helpful in understanding the difference of internal wave characteristics in the ocean. For example, on the Caribbean Coast of Puerto Rico, the internal waves have a distinct seasonal variation as the energy conversion from barotropic tides to internal waves is adjusted by the seasonal variation of stratification [6]; in the Bay of Biscay, the excited internal beams get severely distorted due to their scattering in a summer thermocline, which favors the generation of ISWs [7]; in the northern South China Sea (SCS), due to the asymmetry of stratification on both sides of the Luzon Strait, the characteristics of generated ISWs are also different [8]. Moreover, based on the shallow water Korteweg–deVries (KdV) theory, it can be conjectured that variation in stratification can alter the properties of ISWs [9,10].

There also remains a question regarding the ratio of the wave kinetic energy (KE) to the available potential energy (APE) for a packet of ISWs in different background stratification. For example, observations by Klymak et al. [11] show that, west of the Luzon Strait, the KE is about 40% bigger than the APE for a packet of ISWs; whilst at the Massachusetts Bay, for the nonlinear internal waves before interacting with the bottom topography, the baroclinic energy is thought to be evenly distributed between KE and APE [12]. Theoretically, however, Turkington et al. [13] prove that for exact, fully nonlinear ISWs the KE is always larger than the APE. Based on an accurate formula of APE, Lamb and Nguyen [14] find ratios of KE to APE as high as 30% using quasi-two-layer stratification, and the ratio will increase if the thermocline moves away from the mid-depth. It is also suggested that changes in background stratification influence the conversion of the barotropic-to-baroclinic tide [15].

However, work about the effect of the complex changes in the real oceanic stratification on the ISW characteristic and energy conversion is still relatively unexplored, e.g., the oceanic stratification may have one main thermocline but sometimes two thermoclines, and the depth of thermocline can change largely with time, etc. How do these kinds of variations in vertical stratification affect the ISW characteristic and energy conversion? Thus, a systematic and quantitative study about the effects of different stratification on ISWs is needed. Therefore, an attempt has been made in this paper to study the following two problems. First, how do changes in stratification affect the characteristics of the generated ISWs? Second, how do they affect the partition of the baroclinic energy of ISWs? The full two-dimensional Euler equations model [5], which was employed to the numerical study of the ISWs in the SCS by Xie et al. [16,17], is again used in this study. In the following, the model description, the choice of model parameters and the design of standard stratification are described in Section 2. In Section 3, 9 standard experiments and 140 sensitivity experiments are carried out, and the experimental results are shown and discussed. Finally, the conclusions are summarized in Section 4.

2. Model description and choice of parameters

Considering a two-dimensional (x,z) flow in a stratified ocean, internal waves can be described by the following set of equations,

$$\omega_t + J(\omega,\psi) - f v_z = g\tilde{\rho}_x/\bar{\rho}_0 + A^H\omega_{xx} + \left(A^V\omega_z\right)_z, \tag{2}$$

$$v_t + J(v,\psi) + f\psi_z = A^H v_{xx} + \left(A^V v_z\right)_z, \tag{3}$$

$$\tilde{\rho}_t + J(\tilde{\rho},\psi) + \bar{\rho}_0/gN^2(z)\psi_x = K^H\tilde{\rho}_{xx} + \left(K^V\tilde{\rho}_z\right)_z + \left(K^V\rho_{0z}\right)_z, \tag{4}$$

$$\omega = \psi_{xx} + \psi_{zz}, \tag{5}$$

where ω is the vorticity, ψ is the streamfunction, $\rho_0(z)$ is the stationary density and $N(z)$ is the corresponding buoyancy frequency, $\bar{\rho}_0$ is a constant average of density, $\tilde{\rho}$ is the density disturbance because of the wave motion. The seawater density is written as $\rho(z) = \rho_0(z) + \tilde{\rho}(x,z,t)$. (u,v,w) is the velocity vector, $f = 5.07 \times 10^{-5}\ \mathrm{s}^{-1}$ is the Coriolis parameter. $A^V = 1 \times 10^{-5}\ \mathrm{m}^2\ \mathrm{s}^{-1}$, $A^H = 1 \times 10^{-4}\ \mathrm{m}^2\ \mathrm{s}^{-1}$, $K^V = 1 \times 10^{-5}\ \mathrm{m}^2\ \mathrm{s}^{-1}$ and $K^H = 1 \times 10^{-4}\ \mathrm{m}^2\ \mathrm{s}^{-1}$ are the vertical and horizontal coefficients of turbulent viscosity and mass diffusivity, respectively. J is the Jacobian operator and g is the acceleration due to gravity. It should be noted that the calculation is two-dimensional in the sense that $\partial(\ )/\partial y = 0$, but v is present because of the Coriolis force.

The boundary and initial conditions are given as follows,

$$\psi = 0, \quad \omega = 0, v_{\vec{n}} = 0, \quad \tilde{\rho}_{\vec{n}} = 0, \quad (z = 0), \tag{6}$$

$$\psi = \psi_0 \sin\sigma t, \quad \omega = 0, \quad v_{\vec{n}} = 0, \quad \tilde{\rho}_{\vec{n}} = 0, \quad (z = H(x)), \tag{7}$$

$$\psi = -z\psi_0\sin(\sigma t)/H_0, \quad \omega = 0, \quad v = -f/\sigma\psi_0\cos(\sigma t)/H_0, \quad \tilde{\rho} = 0, \quad (x = -L), \tag{8}$$