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Power laws behavior in multi-state elastic models with different constraints in the statistical distribution of elements

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ABSTRACT

Often materials exhibit nonlinearity and hysteresis in their response to an elastic excitation and the dependence of the nonlinear indicator on the excitation energy is a power law function. From the theoretical point of view, such behavior could be described using multistate elastic models based on a generalized Preisach–Mayergoyz (PM) approach. In these models a statistical distribution of transition parameters is usually introduced. We show in this paper the existence of a link between the power law exponent predicted by the model and the properties of the chosen distribution. Numerical results are discussed, based on an implementation in the PM formalism of an adhesion model.

1. Introduction

In nonlinear elasticity, often nonlinear indicators y are defined and measured as a function of the excitation amplitude x, to estimate and characterize the elastic response of the sample [1,2]. Among them, the most commonly used are the relative resonance frequency shift [3–5], the ratio between amplitudes of the higher order harmonics and fundamental [6,7], the parameters measured in a Dynamic AcoustoElastic Testing [8], the Scaling Subtraction Method indicator [9,10], the ratio of the sidebands amplitude [11], etc. Provided the excitation energies are in a proper range to avoid noise and saturation effects [12], the dependence of y on x could be always described as a power law dependence $y = ax^b$ [1,13]. The determination of the kind and strength of nonlinearity strongly depends on the value of the exponent b of the power law fitting [14,15].

In experiments, if y is defined as an adimensional quantity, the exponent b may assume different values, depending of the nature and intensity of nonlinearity and damage present in the material. In metals, where atomistic nonlinearity is dominant, $b = 2$ is observed [16]; in intact consolidated granular media or samples with closed microcracks, the exponent is $b = 1$ [17–20], while in samples subject to mechanical damage, such as thermal damage, fatigue or quasistatic loadings, b increases up to 3 with increasing the number and extension of open cracks [21–23].

From the theoretical point of view, the equation of state of such systems is usually introduced using discrete models, due to the difficulty to formulate in a continuous and analytical form the description of threshold activated mechanisms or

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memory and hysteresis. The observed variation of b could thus be explained assuming that in different conditions different physical mechanisms are at play, thus requiring different physical models: hysteresis [24–26], adhesion [27–29], clapping [30–32], etc. Indeed, when discrete statistical multistate models are used to describe the system, the exponent b is directly linked to the number of states used [15].

However, an additional point of view should also be considered. When introducing a multistate system, a driving variable controlling transitions from one state to another should be defined. Since multistate models are based on the statistical averaging over elements with different values of the transition parameters [33], it follows that their statistical distribution is also important to characterize the nonlinear response expected by a given model [34]. Furthermore, constraints might be present on the choice of the transition parameters. The classical case is when one of the transition is necessarily smaller than another, for instance in a three state model, where the transition from state 1 to state 2 should occur before the transition from state 2 to state 3. More complex constraints could however be needed, such as an upper bound limit or threshold values.

In this paper, we will show how the exponent of the power law dependence of the nonlinear indicator on the excitation energy could be predicted from the knowledge of the distribution of the transition parameters and of the relative constraints. In our analysis we consider adhesion mechanisms as causing nonlinearity, following a model reported in the literature [29] which is reformulated as a discrete multistate model. However, our conclusions could be easily generalized to other models, such as based on clapping [32] or hysteretic mechanisms [35]. Also, we will analyze two specific nonlinear indicators, based on the generation of higher order harmonics [36] and on the loss of proportionality [37] induced by the presence of nonlinear terms in the wave equation. Similar considerations remain valid also when other nonlinear indicators are considered.

In the next section, we will shortly recall the basic ideas behind the formulation of a multistate model and we will present in details the equations of the specific model considered. In the following Section, the numerical procedure adopted and data analysis will be discussed. Results will be presented in Section 4.

2. Theory

2.1. Multistate models

When effects such as threshold activated mechanisms or memory, conditioning and hysteresis are dominating the elastic response of a medium, analytical models are often difficult to implement. As an alternative, a statistical approach could be more easily adopted based on a generalized Preisach–Mayergoyz approach (PM) [38,39]. In this case, the macroscopic stress–strain equation is obtained from a proper statistical average of the responses of "microscopic" discrete elements (MEs) characterized by a multistate constitutive equation in which transitions from one state to another are controlled by a proper physical variable (stress [35] or strain [40]).

Each ME is defined as a linear spring supporting a force f proportional to the deformation ε with proportionality constant k which might assume different values in the different states (see Fig. 1(a)). If we have N states, $\vec{K} = (k_1, k_2, \ldots k_N)$ is a N-dimensional vector. Thus

$$f = k_i\varepsilon = k_i(l - l_{0i}) \quad \{i = 1, \ldots, N\} \tag{1}$$

where l and l_{0i} are the ME's length and rest-length, respectively.

Given a driving variable z, transitions from state i to state $i+1$ occur when $z = z_{i,i+1}$. Thus a second vector is introduced: $\vec{Z} = (z_{1,2}, z_{2,3}, \ldots z_{N-1,N})$. It follows

$$k = k_i \quad \text{if } z_{i-1,i} < z \leqslant z_{i,i+1} \tag{2}$$

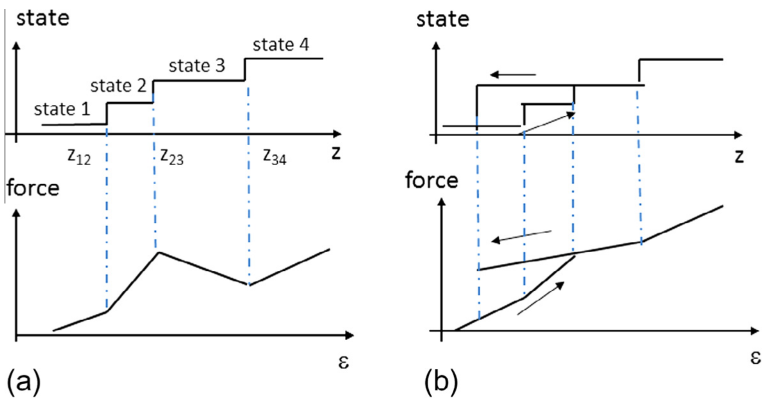

Fig. 1. Schematic description of the definition of multistate discrete models. (a) example of equation of state for the Mesoscopic Element without hysteresis; (b) example of equation of state for the Mesoscopic Element with hysteresis.