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Fault detection based on fractional order models: Application to diagnosis of thermal systems

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ABSTRACT

The aim of this paper is to propose diagnosis methods based on fractional order models and to validate their efficiency to detect faults occurring in thermal systems. Indeed, it is first shown that fractional operator allows to derive in a straightforward way fractional models for thermal phenomena. In order to apply classical diagnosis methods, such models could be approximated by integer order models, but at the expense of much higher involved parameters and reduced precision. Thus, two diagnosis methods initially developed for integer order models are here extended to handle fractional order models. The first one is the generalized dynamic parity space method and the second one is the Luenberger diagnosis observer. Proposed methods are then applied to a single-input multi-output thermal testing bench and demonstrate the methods efficiency for detecting faults affecting thermal systems.

1. Introduction

Heat transfer has many applications in the industrial field. In fact, most forms of energies such as electronic, electromagnetic, radiative, chemical or mechanical degrade into thermal energy. It is common that heat transfers act as constraints to respect and to take into account in order to avoid overheating risks. Indeed, overheating is among the most critical problems since the operating temperature and the thermal cycling can affect the device performance and reliability [1]. In overhead electrical lines for instance, overcoming the maximum admissible temperature of the conductor can limit transmissible power in a transmission line. It causes clearance reduction and conductors rigidity [2]. Moreover, in power electronic devices such as power transistors, CPUs and power diodes, if a heat producing electronic component is considered in isolation then its temperature rises until reaching equilibrium. The rate of loss of heat from a hot object is governed approximately by Newton's law of cooling. This law states that it is proportional to the temperature difference between the body and the surrounding. As the temperature of the component rises, the heat loss increases. When the heat loss per second becomes equal to the heat produced per second within the component, the device achieves its temperature equilibrium. This temperature may be high enough to significantly shorten the life of the component or even cause its failure. Furthermore, hazardous problems can happen in cars due to overheating phenomena. For instance, if the car's thermostat is faulty, the high temperatures can cause the antifreeze to boil, expand, and cause intense pressure within the radiator hoses. This can potentially result in

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hoses suddenly bursting or spraying hot coolant from a weak or broken seal. In addition, many vehicles are equipped with aluminum cylinder heads. However, aluminum is not a material that can withstand a lot of heat without warping or melting. Eventually, the cylinder heads can warp resulting on a blown head gasket. This conflicts with the combustion process as the heads do not perform particularly when they are warped, leading to a decreased engine power, misfiring, oil leaks or excessive oil burning.

A variety of protective measures are studied in the literature to overcome such problems. Most of them are based on thermal modeling [2–4]. Nevertheless, diagnosis procedures are also of high importance in this context. Regardless the nature of measures either protective or diagnostic, a thermal model is needed.

Considering that fractional models have proven their efficiency for modeling thermal diffusion [5–9] and having the aim to contribute in limiting thermal problem risks, this work aims at applying two fractional model based diagnosis methods to a thermal testing bench. The first method is the generalized dynamic parity space method developed in [10–14]. The second method is based on a fractional Luenberger diagnosis observer. The use of fractional Luenberger observer in a diagnosis context is a new approach which has never been proposed in the literature.

This paper is organized as follows. Some definitions and properties of fractional calculus and fractional order models are presented in Section 2. It is demonstrated in Section 3 that thermal systems can be modeled using fractional operators. The structure and the model of the testing bench are described in Section 4. The proposed diagnosis methods are presented in Section 5. Implementation results of these methods are detailed in Section 6. Conclusions are finally given in Section 7.

2. Preliminaries

2.1. Fractional integral and derivative

The Riemann fractional integral of a function $f(t)$ at a positive real number v is defined by [15]:

$$I^v f(t) = \frac{1}{\Gamma(v)} \int_0^t \frac{f(\tau)}{(t-\tau)^{1-v}} d\tau \tag{1}$$

where $\Gamma(v)$ is the Euler function:

$$\Gamma(v) = \int_0^\infty e^{-x} x^{v-1} dx. \tag{2}$$

Contrary to fractional integration, there exist multiple definitions for fractional derivative. Among these definitions, the one used in this work is *Grünwald*'s definition [16] that is the generalization of well known Cauchy's formula for multiple derivatives. *Grünwald*'s fractional derivative of $f(t)$ at non-integer order $v \in \mathbb{R}^+$ is defined as:

$$D^v f(t) = \lim_{h \to 0} \frac{1}{h^v} \sum_{k=0}^{\infty} (-1)^k \binom{v}{k} f(t-kh), \tag{3}$$

where $\binom{v}{k} = \frac{\Gamma(v+1)}{k!\Gamma(v-k+1)}$ is generalized Newton's binomial coefficient.

$D^v f(t)$ can be numerically evaluated using many approximations [17–19]. Using *Grünwald*'s definition and considering that $f(t)$ is null $\forall t \leqslant 0$, fractional derivative can be approximated by:

$$D^v f(t) \approx \frac{1}{h} \sum_{k=0}^{[\frac{t}{h}]} (-1)^k \binom{v}{k} f(t-kh) \tag{4}$$

where h is a sufficiently small constant.

Eq. (4) shows that fractional derivative is not a local operator, because quantity (4) is not zero as $k > v$ and $v \notin \mathbb{N}$, whereas it is zero when $k > v$ and $v \in \mathbb{N}$. Hence, $D^v f(t)$ depends on the whole past of $f(t),\ t \in [0, t]$, unless differentiation order v is integer.

2.2. Fractional systems representation

A Multiple-Input Multiple-Output (MIMO) Linear Time Invariant (LTI) fractional model H, of input $u(t) \in \mathbb{R}^p$ and output $y(t) \in \mathbb{R}^m$, can be described by a fractional differential equation:

$$\sum_{i=0} N_y S_i D^{v_{y_i}} y(t) = \sum_{i=0}^{N_u} T_i D^{v_{u_i}} u(t) \tag{5}$$

where $S_i \in \mathbb{R}^{m \times m}$ and $T_i \in \mathbb{R}^{m \times p}$ are constants matrices, $D^{v_{y_i}}$ and $D^{v_{u_i}}$ are fractional derivative operators.

State space representation was extended in [20] to commensurate fractional models, where all the differentiation orders are multiple integers of v. A fractional state space representation is of the form: