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Synchronization of two coupled multimode oscillators with time-delayed feedback

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ABSTRACT

Effects of synchronization in a system of two coupled oscillators with time-delayed feedback are investigated. Phase space of a system with time delay is infinite-dimensional. Thus, the picture of synchronization in such systems acquires many new features not inherent to finite-dimensional ones. A picture of oscillation modes in cases of identical and non-identical coupled oscillators is studied in detail. Periodical structure of amplitude death and "broadband synchronization" zones is investigated. Such a behavior occurs due to the resonances between different modes of the infinite-dimensional system with time delay.

1. Introduction

Study of synchronization effects in systems of coupled oscillators is an important problem of the modern nonlinear dynamics [1–5]. In particular, a problem of synchronization of time-delayed systems is of great importance because such systems are widespread in neuronal dynamics, nonlinear optics, biophysics, geophysics, telecommunication and information engineering, economics, and ecology [3,6–21]. Systems with time delay are usually described by functional delay-differential equations (DDEs). DDEs are known to have infinite-dimensional phase space [17,18] and are capable of demonstrating a variety of dynamic regimes including chaos [3,9–12,15].

Among the various effects which are observed in systems of interacting oscillators, the amplitude death (AD) effect has been a topic of interest [1,22–29]. AD is a phenomenon of oscillation suppression as a consequence of dissipative coupling. Such a behavior is also characterized by suppression of amplitudes to zero values. AD is desirable in various applications where fluctuations should be suppressed and a constant output is needed. In particular, the amplitude death caused by the time delay in coupling has been studied in many works [25–29]. On the contrary, in [30] it was shown that the delay coupling may induce chaotic behavior, even in a simple system of coupled oscillators.

Features of synchronization of time-delayed systems have been studied in several works. A comprehensive review of the problem has been recently given in [21]. In particular, Usacheva and Ryskin [31] have investigated the forced synchronization of a delayed-feedback oscillator driven by an external harmonic signal. Mensour and Longtin [32] have considered drive-response synchronization of two time-delayed systems with application to secure communication. Ghosh et al. [33]

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have investigated a problem of synchronization between two forced oscillators with unidirectional coupling, when one oscillator has the time delay. In [34], Ghosh et al. have studied a design of delay coupling for targeting desired regime (synchronization, anti-synchronization, lag-synchronization, amplitude death, and generalized synchronization) in mismatched time-delayed dynamical systems.

There exists a great variety of coupling topologies between oscillators with delay. In particular, the networks with time-delayed dissipative linear coupling, as well as pulse-coupled time-delayed networks, have been widely investigated [21]. Many of radio-frequency, microwave, and optical oscillators utilize a power amplifier with input connected with output via a time-delayed feedback loop [11,12,31,35,36]. The most natural way to couple two of such systems is to feed a portion of power from the feedback loop of one oscillator into the feedback loop of another one, and vice versa. In such a case, we have a specific type of coupling, i.e. a nonlinear time-delayed dissipative coupling, which has not been studied previously. In Section 2, we consider a model of coupled oscillators described by a system of coupled DDEs. In Section 3, we study synchronization of two oscillators with identical parameters. Section 4 contains extension to the case of non-identical oscillators. Oscillators with frequency mismatch, as well as with non-identical excitation parameters which determine the oscillation amplitudes, are considered. Transitions between different synchronization regimes are investigated. A special attention is paid to peculiarities of amplitude death and "broadband synchronization" (BS). BS has previously been observed in ensembles of finite-dimensional dissipatively coupled oscillators with non-identical excitation parameters, which are responsible for the oscillation amplitudes [4,37,38]. In such systems there appears a domain of synchronous regimes, which looks like a narrow band located between the AD and quasi-periodic domains and extends to very large values of the frequency mismatch. In the BS domain, the oscillator with larger amplitude dominates and suppresses natural oscillations of the other oscillators.

2. Model and basic equations

2.1. Single delayed-feedback oscillator

Consider a general scheme of a delayed-feedback ring-loop oscillator presented in Fig. 1(a). The oscillator consists of a nonlinear power amplifier, a bandpass filter, and a feedback leg containing a delay line, a variable attenuator, and a phase shifter. The filter is assumed to be narrow-band with Lorenz-shape frequency response. In such a case, it is convenient to express the signal as a quasi-harmonic oscillation with slowly varying amplitude $A(t)$ and carrier frequency ω_c: $A(t)\exp(i\omega_c t)$. It is convenient to choose $\omega_c = \omega_0$ where ω_0 is a central frequency of the filter passband (Fig. 1(a)). Under this assumption, one can derive an equation describing the dynamics of the slow amplitude [11,12,31]

$$\frac{\mathrm{d}A}{\mathrm{d}t} + \gamma A - \alpha f(|A(t-\tau)|)e^{i[\Phi(|A(t-\tau)|)+\theta]} = 0. \tag{1}$$

Here $\gamma = \omega_0/2Q$ is the parameter of losses, Q is the filter Q-factor, $\alpha = \gamma\rho G$ is the parameter of excitation, ρ is the amount of feedback, G is the small-signal gain factor of the amplifier, θ is the phase shift in the feedback loop, f and Φ are nonlinear amplitude and phase transfer functions of the amplifier, respectively, and τ is the delay time.

Further we suppose that the nonlinear amplitude response of the amplifier is approximated by a cubic polynomial and neglect the phase nonlinearity. In such a case, Eq. (1) becomes

$$\frac{\mathrm{d}A}{\mathrm{d}t} + \gamma A - \alpha e^{i\theta}(1 - |A(t-\tau)|^2)A(t-\tau) = 0. \tag{2}$$

Note that without the delay Eq. (2) becomes the well-known normal form for Andronov–Hopf bifurcation [39].

Dynamics of this oscillator has been studied in detail [31,35,36]. It was shown that for a single-frequency solution $A = R_0\exp(i\omega t)$, where R_0 may assumed to be real without loss of generality, the eigenfrequencies obey the equation

$$\omega = -\gamma\tan(\omega\tau - \theta). \tag{3}$$

This equation has infinite number of complex roots, since time-delayed systems with infinite-dimensional phase space have infinite number of eigenmodes. However, only the roots with $\cos(\omega\tau - \theta) > 0$ correspond to stable eigenmodes. Their amplitudes satisfy the equation

$$R_0^2 = 1 - \frac{\sqrt{\gamma^2 + \omega^2}}{\alpha} = 1 - \frac{\alpha_{st}}{\alpha}. \tag{4}$$

These solutions exist when the excitation parameter α exceeds the self-excitation threshold

$$\alpha_{st} = \sqrt{\gamma^2 + \omega^2}. \tag{5}$$

The self-excitation boundary on the θ–α parameter plane has the shape of a periodic set of domains named "generation zones". In the centers of such generation zones at $\theta = 2\pi n$ the frequency $\omega = 0$, i.e. generation arises with frequency equal exactly to the resonance frequency of the filter. Accordingly, the self-excitation threshold is minimal: $\alpha_{st} = \gamma$. At the borders of generation zones at $\theta = 2\pi n + \pi$ two eigenfrequencies are spaced equally from the central frequency. Here, there are