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Dynamics of a driven magneto-martensitic ribbon

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ABSTRACT

We investigate the dynamics of a sinusoidally driven ferromagnetic martensitic ribbon by adopting a recently introduced model that involves strain and magnetization as order parameters. Retaining only the dominant mode of excitation we reduce the coupled set of partial differential equations for strain and magnetization to a set of coupled ordinary nonlinear equations for the strain and magnetization amplitudes. The equation for the strain amplitude takes the form of parametrically driven oscillator. Finite strain amplitude can only be induced beyond a critical value of the strength of the magnetic field. Chaotic response is seen for a range of values of all the physically interesting parameters. The nature of the bifurcations depends on the choice of temperature relative to the ordering of the Curie and the martensite transformation temperatures. We have studied the nature of response as a function of the strength and frequency of the magnetic field, and magneto-elastic coupling. In general, the bifurcation diagrams with respect to these parameters do not follow any standard route. The rich dynamics exhibited by the model is further illustrated by the presence of mixed mode oscillations seen for low frequencies. The geometric structure of the mixed mode oscillations in the phase space has an unusual deep crater structure with an outer and inner cone on which the orbits circulate. We suggest that these features should be seen in experiments on driven magneto-martensitic ribbons.

1. Introduction

It is well known that sinusoidally driven nonlinear oscillators have long been used to model a wide variety of systems ranging from driven nonlinear mechanical oscillators [1,2] to cardiac tissues [3]. While real physical systems may be quite different and complicated, the salient dynamical features are captured by sinusoidally driven model nonlinear oscillators, particularly when it involves only a single order parameter (**OP**) as in models for the p–n junction [4], mechanical oscillator [1,2] and Josephson junction [5]. However, there are several novel materials whose equilibrium properties are controlled by the co-existence of multiple order parameters. Such systems abound in nature. For example, ferroic materials have co-existence of two or more order parameters such as strain, magnetization, electric polarization, etc. [7,8,6]. The application of a field complimentary to one OP induces changes in another OP. For instance, in the case of ferroelectromagnetic materials, the application of an electric field induces magnetization and the application of a magnetic field induces electric polarization. Just as the equilibrium properties are considerably richer than systems with a single OP, one should expect richer dynamics due to enhanced phase space dimension together with the existence of multiple basins of attraction in the phase space that

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are governed by a combination of the minima of the free energies of each of these OPs. Yet, very little attention has been devoted to the dynamical properties of multiple OP systems. The purpose of this paper is to model the dynamics of driven ferromagnetic martensite whose equilibrium or near equilibrium properties has been well studied.

Ferromagnetic martensites [9–15] or simply magneto-martensites as they are called, belong to the class of materials exhibiting strain and magnetization. In particular, these materials exhibit high degree of elastic nonlinearity arising from the twinned nature of the martensite phase, which also couples strongly to magnetization. While most studies have so far been carried out in equilibrium conditions, the strong coupling between the two OPs clearly suggests the possibility of interesting dynamical effects under sinusoidal forcing conditions. However, to the best of authors knowledge, there has been no time dependent dynamical, either experimental or theoretical studies of these materials. This is surprising in view of the well known fact that even simple forced nonlinear mechanical systems exhibit complex dynamical response [1,2]. This is even more surprising since detailed experimental studies on sinusoidally driven magnetostrictive metallic glass ribbons that have significantly lesser degree of nonlinearity compared to ferromagnetic martensites have been reported almost twenty years ago [16–18].

Recently, we developed a coupled OP model that involves strain and magnetization as OPs in an effort to explain the rich spectrum of dynamical features reported on magnetostrictive metallic glass ribbons [19,20]. The modeled features were period doubling and quasiperioidicity routes to chaos, suppression and shift of the onset of chaos under the combined action of a sinusoidal and dc magnetic fields [16–18]. Further, the derived equations of motion were general enough to explain several other unexplained experimental results scattered in the literature on driven ribbons with elastic and magnetic nonlinearities [1,2]. The purpose of this paper is two fold. Since the model equations in Refs. [19,20] are quite general, we adopt them for describing the dynamics of a driven ferromagnetic martensite ribbon. Second, in doing so, we examine the richness of the dynamics of the model equations with respect to several physically relevant parameters which was not investigated in Refs. [19,20]. Moreover, the elastic nonlinearity in magneto-martensites is significantly stronger than that in magentostrictive metallic glasses which already exhibits rich spectrum of bifurcations [16–18]. This is yet another motivation for examining the driven magneto-martensitic ribbon.

We begin by briefly summarizing the physical properties of magneto-martensites with a view to support the relevance of the proposed study. Ferromagnetic martensites have attracted considerable attention for the last three decades due to their technological importance [9–15]. Such alloys (for example Ni_2MnGa, Co_2MnGa, Fe_7Pd_3, etc.) exhibit reversible changes in shape and strain under the application of a magnetic field, and are therefore called magnetic shape memory alloys. These materials undergo a first-order martensitic transformation on cooling and are also ferromagnetic. During the martensitic transformation, the higher symmetry parent (austenite) phase undergoes a spontaneous deformation to a lower symmetry twinned phase. In many of these alloys, the short crystallographic axis is always the easy direction of magnetization [9–15]. This feature coupled with twinning in the martensitic phase and a large magnetic anisotropy implies that large strains can be induced by the redistribution of twin variants caused by the application of even modest strengths of magnetic field. Strain as high as $\sim$6–7% have been reported in ferromagnetic martensites which suggests strong coupling between strain and magnetization [14,21–24]. Several of these features have been captured by a recent two dimensional martensite model that appropriately couples the spin OP to elastic degrees of freedom [25,26]. Indeed, it is this coupling between strain and magnetization that makes them good candidates for actuator applications.

Interestingly, driven martensitic ribbon with just the elastic strain has been studied in the context of internal friction studies of nonmagnetic martensite (CuAlNi) samples in the neighborhood of martensite start temperature T_m where there is a strong elastic nonlinearity with a view to understand the influence of nonlinear strain on the internal friction peak [27,28]. Similar nonlinear response has also been observed in zinc [29] that is attributed to twinning arising from stress. Internal friction experiments are traditionally carried out to measure the anelastic response of the system under low amplitude sinusoidal forcing conditions. The nature of the response depends on the inherent time scales of relaxation arising from the motion of specific types of defects due to application of stress, here twin boundary movement. The temperature dependent frequency response essentially determines the nature of the defect contributing to the internal friction [30]. The results of such measurements are attempted within the scope of *linear* anelasticity, which is obviously inadequate.

Surprisingly, no dynamical studies (sinusoidally driven or internal friction) on magneto-martensites have been reported in the literature in spite of well established strong elastic and magnetic nonlinearities. Our purpose is to carry out model studies on sinusoidally driven magneto-martensitic ribbons. Clearly, a minimal model for magneto-martensite should describe both the martensite phase and ferromagnetic phase so that the elastic and magnetic nonlinearities are properly represented. Such a theoretical study may also suggest experiments to validate the results of the model.

The model equations for magneto-martensites show that the elastic degree of freedom is parametrically driven by the magnetic degree of freedom which itself is excited sinusoidally by an applied magnetic field. We have studied the model as a function of several physically relevant parameters such as the amplitude of the applied magnetic field, the forcing frequency and magneto-elastic coupling. Our analysis shows that the model equations exhibit rich dynamics as a function of each of these parameters. Mixed mode oscillations are seen for low frequencies when the amplitude of the magnetic field crosses a threshold value.