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Research on synchronization of chaotic delayed neural networks with stochastic perturbation using impulsive control method

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ABSTRACT

In this paper, an impulsive controller is designed to achieve the exponential synchronization of chaotic delayed neural networks with stochastic perturbation. By using the impulsive delay differential inequality technique that was established in recent publications, several sufficient conditions ensuring the exponential synchronization of chaotic delayed networks are derived, which can be easily checked by LMI Control Toolbox in Matlab. A numerical example and its simulation is given to demonstrate the effectiveness and advantage of the theory results.

1. Introduction

In the past decades, impulse as an effective control method has been widely used to many fields such as orbital transfer of satellite [1,2], dosage supply in pharmacokinetics [3], ecosystems management [4,5], synchronization in chaotic secure communication systems [6–8] and so on. Its necessity and importance lie in that, in many cases, a real system may encounter some abrupt changes at certain time moments and cannot be considered continuously. Moreover, sometimes even only impulsive control can be used for control purpose. For instance, a central bank can not change its interest rate everyday in order to regulate the money supply in a financial market [9,10]. The basic theory for impulsive differential systems, especially for impulsive delay differential systems, has been extensively investigated in the past several years, see [9–16] and the references therein. In particular, Yan and Shen [11] studied the impulsive stabilization of functional differential equations with finite delays by using Lyapunov–Razumikhin method; Liu [12] investigated the stability for a class of linear delay systems via impulsive control; Li [14] considered the global exponential stabilization of impulsive functional differential equations with infinite delays or finite delays by using the improved Lyapunov–Razumikhin method.

On the other hand, it has been shown that delayed neural networks can exhibit some complicated dynamics and even chaotic behaviors if the network's parameters and time delays are appropriately chosen [17,18] and many interesting synchronization schemes of chaotic delayed neural networks have been proposed via different approaches, see [18–23] and the

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references therein. For instance, Song [19] considered the synchronization problem of chaotic neural networks with mixed time-varying delays via time-delay feedback approach. In [21], Park studied the synchronization problem of cellular neural networks of neutral type via dynamic feedback approach. In particular, impulsive control method for synchronization problem of chaotic systems has been studied and developed in recent years [24–27]. Its extraordinary superiority is that, using the impulsive control, the response system receives the signals from the drive system only in the impulsive instances and thus the amount of conveyed information is decreased, which can not only increase robustness against the disturbances but also greatly reduce the control cost. Now there are some but very little studies in the synchronization issue of chaotic delayed neural networks using impulsive control method [28–31] due to the limits of research techniques. In [28], Li et al. studied the robust impulsive control synchronization of coupled delayed neural networks with uncertainties by utilizing the stability theory developed in [11]. In [29], Zhang and Sun considered the robust impulsive synchronization of coupled delayed neural networks using the analysis theory of impulsive functional differential equations. Recently, Li [30,31] investigated the complete synchronization and lag synchronization of chaotic delayed neural networks via the stability criteria developed in [14]. However, one may note that the existing results [28–31] for chaos synchronization via impulsive control are only based on the basic theory for impulsive functional differential systems, which cannot be applied to synchronization problem of chaotic neural networks with stochastic perturbation. Noises are ubiquitous in both nature and man-made systems such as, in the real nervous systems, synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes [32]. Recently, some interesting results on stochastic effects to the synchronization problem of delayed neural networks have been reported, see [33–36]. Since it has been stated in the above that impulsive control method has its extraordinary superiority, one may consider whether we can establish some results on synchronization problem of delayed neural networks with stochastic perturbation using impulsive control method. Unfortunately, due to some theoretical and technical difficulties, up to now the chaos synchronization of delayed neural networks with stochastic perturbation via impulsive control has not been addressed, which is still an open problem and remains challenging.

Inspired by the above discussion, in the paper, we will discuss the synchronization problem of chaotic delayed neural networks with stochastic perturbation via impulsive control method. Moreover recently, Li and Bohner [38] proposed a new impulsive delay differential inequality from impulsive control point of view. Base on the result, in this paper we will derive an impulsive controller for exponential synchronization of chaotic delayed neural networks with stochastic perturbation. The rest of this paper is organized as the following. Some preliminaries are provided in Section 2. In Section 3, some sufficient conditions for the exponential synchronization are derived by constructing the suitable impulsive controller. In Section 4, a numerical example and its simulation is given to demonstrate the effectiveness of the theory results. Conclusions are given in Section 5.

2. Preliminaries

Notations. Let $\mathbb{R}$ denotes the set of real numbers, $\mathbb{Z}_+$ denotes the set of positive integers, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denotes the n-dimensional and $n\times m$-dimensional real spaces equipped with the Euclidean norm $|\bullet|$, respectively. $E(\cdot)$ stands for the mathematical expectation of a stochastic process. $\mathscr{A}>0$ or $\mathscr{A}<0$ denotes that the matrix $\mathscr{A}$ is a symmetric and positive definite or negative definite matrix. $\mathscr{A}\geqslant 0$ or $\mathscr{A}\leqslant 0$ denotes that the matrix $\mathscr{A}$ is a symmetric and positive semidefinite or negative semidefinite matrix. The notation $\mathscr{A}^T$ and $\mathscr{A}^{-1}$ mean the transpose of $\mathscr{A}$ and the inverse of a square matrix. If $\mathscr{A},\mathscr{B}$ are symmetric matrices, $\mathscr{A}>\mathscr{B}$ means that $\mathscr{A}-\mathscr{B}$ is a positive definite matrix. $\lambda_{\max}(\mathscr{A})$ or $\lambda_{\min}(\mathscr{A})$ denotes the maximum eigenvalue of matrix $\mathscr{A}$ or the minimum eigenvalue of matrix $\mathscr{A}$. For any interval $J\subseteq\mathbb{R}$, set $S\subseteq\mathbb{R}^k(1\leqslant k\leqslant n), C(J,S)=\{\varphi:J\to S \text{ is continuous}\}$ and $PC(J,S)=\{\varphi:J\to S$ is continuous everywhere except at finite number of points t, at which $\varphi(t^+)$, $\varphi(t^-)$ exist and $\varphi(t^+)=\varphi(t)\}$. I denotes the identity matrix with appropriate dimensions and $\Lambda=\{1,2,\ldots,n\}$. Moreover, the notation $\star$ always denotes the symmetric block in one symmetric matrix.

Consider the following recurrent network:

$$\begin{cases} dx(t)=[-Cx(t)+Af(x(t))+Bf(x(t-\tau(t)))+J]dt, & t>0,\\ x(s)=\phi(s), & s\in[-\tau,0], \end{cases} \tag{1}$$

where $x(t)=(x_1(t),\ldots,x_n(t))^T$ is the neuron state vector of the neural network and $n\in\mathbb{Z}_+$ denotes the number of neurons in the network; $C=\text{diag}(c_1,\ldots,c_n)$ is a diagonal matrix with $c_i>0, i\in\Lambda$; A,B are the connection weight matrix and the delayed connection weight matrix, respectively; $f(x(\cdot))=(f_1(x_1(\cdot)),\ldots,f_n(x_n(\cdot)))^T$ represents the neuron activation function; $\tau(t)$ is the transmission delay and J is an external input; $\phi(\cdot)\in C([-\tau,0],\mathbb{R}^n)$.

We consider network (1) as the master system. The response one is given by

$$\begin{cases} dy(t)=[-Cy(t)+Af(y(t))+Bf(y(t-\tau(t)))+J]dt+\sigma(t,e(t),e(t-\tau(t)))d\omega(t), & t\in[t_{k-1},t_k),\\ \Delta y(t_k)=y(t_k)-y(t_k^-)=-Hu(t_k^-), & k\in\mathbb{Z}_+,\\ y(s)=\varphi(s), & s\in[-\tau,0], \end{cases} \tag{2}$$

where model (2) has the similar structure as the drive system (1); $\omega(t)=(\omega_1(t),\ldots,\omega_m(t))^T$ is an m-dimensional Brownian motion defined on a complete probability space $(\Omega,\mathscr{F},\mathscr{P})$ with a natural filtration $\{\mathscr{F}_t\}_{t\geqslant 0}$ generated by $\{\omega(s):0\leqslant s\leqslant t\}$,