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Effect of induced magnetic field on peristaltic transport of a Carreau fluid

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ABSTRACT

Magnetohydrodynamic (MHD) peristaltic flow of a Carreau fluid in a channel with different wave forms are analyzed in this investigation. The flow analysis is conducted in the presence of an induced magnetic field. Long wavelength approach is adopted. Mathematical expressions of stream function, magnetic force function and an axial induced magnetic field are constructed. Pressure rise and pumping phenomena are described.

1. Introduction

Peristaltic motion in recent times has received considerable attention due to its applications in physiological fluids such as vasomotion of small blood vessels, chyme motion in the gastrointestinal tracts, sperm transport in the ductus efferentus of the male reproductive tract, movement of ovum in the fallopian tube, swallowing of food through esophagus etc. The principle of peristaltic transport is also exploited in many industrial applications. Sanitary fluid transport, transport of corrosive fluids and blood pumps in heart lung machines are few of these. Since the seminal work of Latham [1], many investigations dealing with peristaltic flows under different flow geometries and assumptions have been presented by employing analytical, numerical and experimental approaches. Few recent studies in this direction may be mentioned [2–14]. However, the influence of an induced magnetic field on the peristaltic transport is not examined much. To the best of our information, Mekheimer [15,16] studied the peristaltic flow of couple stress and micropolar fluids in a symmetric channel when an induced magnetic field is taken into account. Hayat et al. [17] presented the study of peristaltic flow of a third order fluid in the presence of an induced magnetic field.

The aim of this communication is to examine the influence of an induced magnetic field on the Carreau fluid. The flow is ergendered by progressive waves on the non-conducting channel walls. This paper runs in the following arrangement. Section two deals with the problem formulation in the non-dimensional variables. In section three, series solutions for small Weissenburg number is developed. Expressions of different wave shapes are given in section four. Section five includes the discussion.

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2. Development of the mathematical problem

Let us investigate the MHD flow of an incompressible Carreau fluid in a two dimensional channel of uniform thickness. Four possible wave forms namely sinusoidal (s), triangular (t), square (sq), trapezoidal (tr) travelling down on the channel walls are considered. We consider a wave of amplitude b that propagates on the channel walls with constant speed c. Its instantaneous height at any axial station X' is

$$Y' = H\left(\frac{X' - ct'}{\lambda}\right). \tag{1}$$

It is further noticed that flow in laboratory (X', Y') and wave (x', y') frames are treated unsteady and steady, respectively. The transformations between the two frames are

$$\begin{aligned} &x' = X' - ct', \quad y' = Y', \\ &u'(x', y') = U' - c, \quad v'(x', y') = V', \end{aligned} \tag{2}$$

in which (U', V') and (u', v') are the respective velocities in the laboratory and wave frames. A constant magnetic field of strength H_0' is applied in the transverse direction. This gives rise to an induced magnetic field $H'(h'_{X'}(X', Y', t'), h'_{Y'}(X', Y', t'), 0)$ and hence the total magnetic field is $H'^{+}(h'_{X'}(X', Y', t').\ H_0' + h'_{Y'}(X', Y', t'), 0)$.

An extra stress tensor $\bar{\tau}$ in a Carreau fluid is

$$\bar{\boldsymbol{\tau}} = -\left[\eta_0 (1 + (\Gamma \bar{\dot{\gamma}})^2)^{\frac{n-1}{2}}\right]\bar{\dot{\gamma}}, \tag{3}$$

$$\bar{\dot{\gamma}} = \sqrt{\frac{1}{2}\sum_i \sum_j \bar{\dot{\gamma}}_{ij}\bar{\dot{\gamma}}_{ji}} = \sqrt{\frac{1}{2}\pi}, \tag{4}$$

where η_0 is zero shear-rate viscosity, Γ the time constant, n the dimensionless power law index, π the second invariant of strain-rate tensor and infinite shear stress viscosity is absent. When $n = 1$ or $\Gamma = 0$; Eq. (3) reduces to a Newtonian fluid. The relevant equations are

$$\nabla \cdot \mathbf{H}' = 0, \quad \nabla \cdot \mathbf{E}' = 0, \tag{5}$$

$$\nabla \Lambda \mathbf{H}' = \mathbf{J}', \quad \text{with } \mathbf{J}' = \sigma\{\mathbf{E}' + \mu_e(\mathbf{V}' \Lambda \mathbf{H}'^{+})\}, \tag{6}$$

$$\nabla \Lambda \mathbf{E}' = -\mu_e \frac{\partial \mathbf{H}'}{\partial t'}, \tag{7}$$

$$\nabla \cdot \overline{\mathbf{V}'} = 0, \tag{8}$$

$$\rho\left[\frac{\partial}{\partial t'} + (\mathbf{V}' \cdot \nabla)\right]\overline{\mathbf{V}}' = -\nabla p' + \text{div}\bar{\tau}' - \mu_e\left\{(\mathbf{H}'^{+} \cdot \nabla) - \frac{1}{2}(H'^{+})^2 \nabla\right\}. \tag{9}$$

In the above expressions p' is the fluid pressure, $\mathbf{J}$ the current density, μ_e magnetic permeability, σ the electrical conductivity, $\mathbf{E}'$ an induced magnetic field and the velocity $\overline{\mathbf{V}'}$ is

$$\overline{\mathbf{V}}' = (u', v', 0). \tag{10}$$

Employing Eqs. (5)–(7), the induction equation takes the form

$$\frac{\partial \mathbf{H}'^{+}}{\partial t'} = \nabla \Lambda\{\mathbf{V}' \Lambda \mathbf{H}'^{+}\} + \frac{1}{\zeta}\nabla^2 \mathbf{H}'^{+} \tag{11}$$

in which $\zeta = 1/\sigma\mu_e$ is the magnetic diffusivity. Writing

$$\begin{aligned} &x = \frac{2\pi x'}{\lambda}, \quad y = \frac{y'}{a}, \quad u = \frac{u'}{c}, \quad v = \frac{v'}{c}, \quad t = \frac{2\pi t' c}{\lambda}, \quad p = \frac{2\pi a^2 p'}{c\lambda\mu}, \\ &\tau = \frac{a\tau'}{\mu c}, \quad h = \frac{h'}{a}, \quad \Psi = \frac{\Psi'}{ca}, \quad \phi = \frac{\phi'}{H_0 a} \quad \delta = \frac{a}{\lambda}, \quad Re = \frac{\rho c a}{\mu}, \\ &R_m = \sigma\mu_e a c, \quad S_t = \frac{H_0}{c}\sqrt{\frac{\mu_e}{\rho}} \end{aligned} \tag{12}$$

Eqs. (9) and (11) give

$$Re\delta\left\{\left(\Psi_y \frac{\partial}{\partial x} - \Psi_x \frac{\partial}{\partial y}\right)\Psi_y\right\} = -\frac{\partial p_m}{\partial x} + \delta\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + Re\delta S_t^2\left(\phi_y \frac{\partial}{\partial x} - \phi_x \frac{\partial}{\partial y}\right)\phi_y + ReS_t^2 \phi_{yy}, \tag{13}$$

$$-Re\delta^3\left\{\left(\Psi_y \frac{\partial}{\partial x} - \Psi_x \frac{\partial}{\partial y}\right)\Psi_x\right\} = -\frac{\partial p_m}{\partial y} + \delta^2\frac{\partial \tau_{xy}}{\partial x} + \delta\frac{\partial \tau_{yy}}{\partial x} - Re\delta^3 S_t^2\left(\phi_y \frac{\partial}{\partial x} - \phi_x \frac{\partial}{\partial y}\right)\phi_x - Re\delta^2 S_t^2 \phi_{xy}, \tag{14}$$

$$\Psi_y - \delta(\Psi_y \phi_x - \Psi_x \phi_y) + \frac{1}{R_m}\nabla^2 \phi = E, \tag{15}$$