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Homotopy Analysis Method for the heat transfer of a non-Newtonian fluid flow in an axisymmetric channel with a porous wall

M. Esmaeilpour [a,b], G. Domairry [a,b,*], N. Sadoughi [a,b], A.G. Davodi [c]

[a] Department of Mechanical Engineering, Babol University of Technology, P.O. Box 484, Babol, Iran
[b] Department of Mechanical Engineering, University of Kashan, Kashan, Iran
[c] Department of Civil Engineering, Shahrood University of Technology, Shahrood, Iran

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ABSTRACT

In this article, a powerful analytical method, called the Homotopy Analysis Method (HAM) is introduced to obtain the exact solutions of heat transfer equation of a non-Newtonian fluid flow in an axisymmetric channel with a porous wall for turbine cooling applications. The HAM is employed to obtain the expressions for velocity and temperature fields. Tables are presented for various parameters on the velocity and temperature fields. These results are compared with the solutions which are obtained by Numerical Methods (NM). Also the convergence of the obtained HAM solution is discussed explicitly. These comparisons show that this analytical method is strongly powerful to solve nonlinear problems arising in heat transfer.

1. Introduction

Recently, considerable attention has been devoted to the problem of non-Newtonian fluid flow because of its various applications in different fields of engineering. In particular the interest in heat transfer problem of non-Newtonian fluid flow has grown considerably. Hot rolling, extrusion of plastics, flow in journal bearings, lubrication and flow in a shock absorber are some typical examples to name. Understanding the nature of channel flow of non-Newtonian fluid and related heat transfer problem by mathematical modeling with a view to predict the temperature distribution and the associated behavior of fluid flow have been the focus of considerable research works [1–8].

Since there are some limitations with the common perturbation methods, and also because the basis of the common perturbation method is upon the existence of a small parameter, developing the method for different applications is very difficult. Therefore, many different methods have recently introduced to eliminate the small parameter. The Homotopy Analysis method (HAM) is one of the well-known methods to solve the nonlinear equations. This method has been first introduced in 1992 by Liao [9–14]. The method has been used by many authors [15–22] in a wide variety of scientific and engineering applications to solve different types of governing differential equations: linear and nonlinear, homogeneous and non-homogeneous, and coupled and decoupled as well.

In this Letter, the basic idea of the HAM method is introduced and then, the nonlinear equations of the non-Newtonian fluid flow in a porous channel with high mass transfer are solved through the Homotopy Analysis Method. The organization of this paper is as follows: Section 2 contains the description of the problem and the governing equations have been pre-

* Corresponding author. Address: Department of Mechanical Engineering, Babol University of Technology, P.O. Box 484, Babol, Iran. Tel./fax: +98 111 3234201.
E-mail address: amirganga111@yahoo.com (G. Domairry).

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sented. In Section 3, we simply provide the mathematical framework of the Homotopy Analysis Method and in Section 4 we use these analytical methods to solve the nonlinear equation governing the described problem. Finally, some results are provided.

2. Mathematical formulation

This study is concerned with simultaneous development of flow and heat transfer for non-Newtonian viscoelastic fluid flow on the turbine disc for cooling purposes. The problem to be considered is depicted schematically in Fig. 1. The x-axis is parallel to the surface of disk and the y-axis is normal to it. The porous disc of the channel is at $y = +L$. The wall that coincides with the x-axis is heated externally and from the other perforated wall non-Newtonian fluid is injected uniformly in order to cool the heated wall.

We suppose that the flow field may be assumed to be stagnation point flow with injection. For the non-Newtonian fluid flow that is steady, axisymmetric and two-dimensional, Kurtcebe and Erim have presented the equations which govern the flow and heat transfer as [8]:

$$-2ff''' = \frac{f''''}{Re} - K_1(4f''f''' + 2f'f'''') - K_2(4f''f''' + 2f'f'''' + 2ff'''''), \tag{2.1}$$

$$nf'q_n - 2fq_n' = \frac{1}{Pr \cdot Re} q_n'', \quad (n = 0, 2, 3, 4, \ldots). \tag{2.2}$$

where $K_1 = \frac{\phi_2}{\rho L^2}, K_2 = \frac{\phi_3}{\rho L^2}$, Re is the injection Reynolds number and Pr is Prandtl number. The boundary conditions are:

$$f(0) = 0, \quad f'(0) = 0, \quad f(1) = 1, \quad f'(1) = 0, \tag{2.3}$$

$$q(0) = 1, \quad q(1) = 0. \tag{2.4}$$

Eqs. (2.1) and (2.2) with the boundary conditions (2.3) and (2.4) are solved by Kurtcebe and Erim [8] for $K_2 = 0$. In this article, we reconsider these equations as:

$$f'''' + 2Reff''' - K_1Re(4f''f''' + 2f'f'''') = 0, \tag{2.5}$$

$$q_n'' - Pr \cdot Re(nf'q_n - 2fq_n') = 0, \tag{2.6}$$

and solve them by Homotopy Analysis Method.

3. Basic idea of HAM

Let us assume the following nonlinear differential equation in form of:

$$N[u(\tau)] = 0, \tag{3.1}$$

where N is a nonlinear operator, τ is an independent variable and $u(\tau)$ is the solution of equation. We define the function, $\phi(\tau, p)$ as follows:

$$\lim_{p \to 0} \phi(\tau, p) = u_0(\tau), \tag{3.2}$$

where, $p \in [0, 1]$ and $u_0(\tau)$ is the initial guess which satisfies the initial or boundary condition,

$$\lim_{p \to 1} \phi(\tau, p) = u(\tau), \tag{3.3}$$

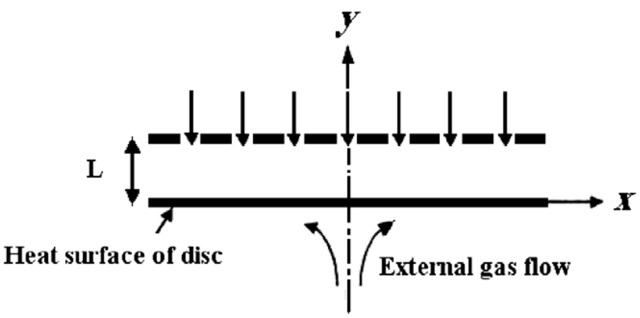

Fig. 1. Schematic diagram of the physical system.