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Numerical simulation of the soft contact dynamics of an impacting bilinear oscillator

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ABSTRACT

Systems constituted by moving components that make intermittent contacts with each other can be modelled by a system of ordinary differential equations containing piecewise linear terms. We consider a soft impact bilinear oscillator for which we obtain bifurcation diagrams, Lyapunov coefficients, return maps and phase portraits of the response. Besides Lyapunov coefficients diagrams, bifurcation diagrams are represented in terms of both non-dimensional time instants of contact (when the mass impacts the obstacle) and of portions of contact duration (the percentage-time interval when the material point is inside the obstacle) vs. non-dimensional external force frequency (or amplitude). The second kind of diagrams is needed because the contact duration (or the complementary flight time duration) are quantities that can easily be measured in an experiment aiming at confirming the validity of the present model. Lyapunov coefficients are evaluated converting the piecewise linear system of ordinary differential equations into a map, the so-called impact map, where time and velocity corresponding to a given impact are evaluated as functions of time and velocity corresponding to the previous impact. Thus, the usual methods related to this last map are used. The trajectories are represented in terms of return maps (all points in the time-velocity plane involved in the impact events) and phase portraits (the trajectory-itself in the displacement-velocity plane). In the bifurcation diagrams, transition between different responses is evidenced and a perfect correlation between chaotic (periodic) attractors and positive (negative) values of the maximum Lyapunov coefficient is found.

1. Introduction

Significant research efforts have been spent in the theory and application of nonlinear dynamics for non-smooth systems [1–6]. Among the wide range of nonlinear dynamical systems, Piecewise Smooth Systems (PSS) play an important role and can be classified as continuous or discontinuous PSS [5,7]. The most simple discontinuous PSS is the impact oscillator studied since 1958 for the case of an electronic bell [8] and also investigated in recent theoretical and numerical works, see e.g. [9–14]. The simplest continuous PSS is the piecewise (or bilinear) oscillator, that is the object of an important work by Shaw and Holmes [15] and is also recently analyzed in many papers, see e.g. [16,17].

Both hard and soft impacts have been considered. The exact solution of the fundamental periodic motion of a simple mechanical system with hard impact, attached to a sinusoidally excited primary mass of the system, was derived analytically by Blazejczyk-Okolewska and Peterka [18]. Blazejczyk-Okolewska et al. [19] determined the regions of periodic motions with impacts and the stability of periodic solutions of a two-degree-of-freedom mechanical system; impacts between the mass

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and the rigid basis were described by a coefficient of restitution. Peterka and Tondl [20] dealt with the analysis, via numerical simulations, of the motion of one degree-of-freedom mechanical system with soft impact; the restoration force is characterized by a triangle hysteresis loop and the external excitation is an harmonic force. Subharmonic impact motions are characterized in a subregion of the plane of dimensionless excitation frequency and static clearance, showing the regions of different regimes of impact motions.

Recently, various types of soft impacting systems have been addressed by Ma et al. [21] via experimental and/or numerical approaches, in the framework of a two-dimensional mapping analysis of the relevant dynamics. Among authors using sophisticated analytical mappings in the study of piecewise systems we mention Luo [22] and Pavlovskaia and Wiercigroch [23].

In this work, we also deal with a soft impact bilinear oscillator which, besides its own interest, also aims at representing a Single Degree Of Freedom (SDOF) model of the first mode of vibration of an impacted cantilever beam of uniform mass, experimentally studied in [11]. The target is to find the conditions for which non-trivial impacting chaotic and periodic motions do occur. To this aim, we use impact maps, whose limitations for studying grazing and low-velocity impacts do not come into play. In order to grossly distinguish between a periodic and a chaotic trajectory, we use Lyapunov coefficients. Lyapunov numbers (exponents or coefficients) measure the average divergence of nearby trajectories. A chaotic system is generally defined under the condition that the associated largest Lyapunov coefficient is positive [24].

For dynamical systems described by smooth differential equations and for discrete maps, the calculation of Lyapunov coefficients is well developed [25]. For non-smooth dynamical systems we have the work of Müller [26] that generalizes classical techniques for smooth dynamical systems, but also the papers of Stefanski [27], who uses chaos synchronization, and Galvanetto [28], who uses the definition of a smooth transcendental map. This map is defined in such a way the event of a certain discontinuity in the solution of the non-smooth dynamical system is given in terms of the foregoing discontinuity and it is also used by de Souza and Caldas [13]. In [13], the authors apply the usual method developed in [25] for discrete maps to this new transcendental map. They apply this idea to the case of an impact oscillator and to the impact pair system. A novelty of the present work lies in the application of the technique used by de Souza and Caldas to the case of an impacting bilinear oscillator. We remark that the tricky point of this application is that the transcendental maps to be defined are two: one map for each smooth region of the problem, namely the contact and the flight regions. Moreover, we study the stability of the trajectories by using the Jacobians in numerical form calculated at the time instants of the attachment and detachment of the material point at the obstacle, as well as the Lyapunov coefficients.

This means that it is not possible to analyze the stability of a period one point through the evaluation of its eigenvalues, see [14], without an empirical assumption on the periods of such trajectories, see e.g. [15]. This is the reason why we will evaluate numerically the stability of the trajectories by using a proper form of the Jacobian.

In order to pursue the analogy with an impacted cantilever beam, not only the springs have two different rigidities in the two smooth regions of the problem, but also the dampers have different attenuation coefficients. The spring and the damper of the system simulate the cantilever beam; the spring and the damper modelling the obstacle have larger values of rigidity (in order to simulate the hardness of the obstacle) and attenuation coefficient (in order to simulate the energy dispersion during the impact event). We remark that, in the impact oscillator model, the hardness of the obstacle and the energy dispersion of the impact event are modelled, respectively, by the fact that the latter occurs instantaneously and by a single restitution coefficient lower than one.

The paper is organized as follows. In Section 2 we present the dimensional and non-dimensional sets of piecewise ordinary differential equations that we treat in the rest of the paper. In Section 3 we describe the iterative solution method and derive the transcendental maps and their Jacobians. In Section 4 the Lyapunov coefficients are described along with the numerical method for their computation. In Section 5 we present a detailed parametric analysis of system non-trivial impact-response through return maps and bifurcation diagrams, and characterize some relevant transition scenarios. Our conclusions end the paper in Section 6.

2. System description

2.1. Dimensional equations

The piecewise linear oscillator considered in this paper is a mass-spring-damper system governed by the following ordinary differential equation,

$$m\ddot{x} + c(x)\dot{x} + k(x) = F_0 \sin(\omega t), \tag{1}$$

where x is the displacement, from the unstressed configuration of the spring with rigidity k_s, of a material point with mass m at time t, the dot denotes the derivative with respect to time, $F_0 \sin(\omega t)$ is the external sinusoidal force; $c(x)$ is the attenuation coefficient and $k(x)$ is the opposite of the force exerted by the system springs. In order to simulate the occurrence of the impact event, $c(x)$ and $k(x)$ are two piecewise functions of x defined as follows,

$$k(x) = \begin{cases} k_s x, & x < \delta, \\ k_s x + k_o(x - \delta), & x \geqslant \delta, \end{cases} \tag{2}$$